STATISTICS AS AN AID TO THE COMPTROLLER

by

S. Frank Leis

Major USMC

for

Doctor A. Rex Johnson

The George Washington University

Navy Graduate Comptrollership Program

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"The Time may not be very remote when it will be understood that for complete initiation as an efficient citizen of one of the new great complex world wide states that are now developing, it is as necessary to be able to compute, to think in averages and maxima and minima, as it is now to be able to read and to write."

--H. G. Wells,
Mankind in the Making.

PREFACE

The success of statistical techniques in improving the quality of manufactured goods has led to the adoption of these techniques into non-manufacturing fields. Today these highly developed mathematical techniques are providing case histories of successes in overcoming problems concerned with the improvement of goods, services, and performance through better control methods, and through analysis of data in decision making. Statistics is commonplace in the fields of stock market, advertising, sales surveys, and accounting. The statistical methods based on the law of probability, which were successful in controlling war-production quality, are now being applied to service, staff, and administrative activities.

Although there are many favorable reports and case histories to substantiate the value of the application of statistical technique to decision making there are many who do not accept these methods. Statistics are "dry" and there is mystery in figures and formula. There are many who distrust figure people. Statistics is a scare word and hence must be sold to be accepted.

It is the purpose of this paper to provide, in as far as is possible in dealing with a subject of this sort, a nontechnical discussion of these statistical techniques. It

ABSTRACT

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It is the purpose of this paper to present, in so far as is possible [illegible] mathematical discussion of [illegible] statistical techniques. It is not the purpose here to do more than merely scratch the surface of statistical theory to provide only what is necessary to demonstrate the application of statistical method in solving rather routine problems. It is not necessary to become a skilled mathematician, the technical phases of statistical method can be delegated to the technicians--if it is clearly understood at the management level what the objectives are and if proper support is given the staff. Although the comptroller need not know detailed technical applications, he should be interested and have developed some feel and appreciation for such methods. The use of statistical and mathematical method is in reality a front-office tool and the higher one ascends within the management structure the more important it becomes as a management technique of analysis and control.

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CHAPTER I

STATISTICS AND THE COMPTROLLER

Introduction

Statistics is the offspring of a varied ancestry. As a result of greed the ancient kings enumerated their people for the purpose of taxation; the panic of an English Sovereign during the London plague; the cupidity of professional gamblers; the scientific ardor of the psycho-physicists; the labors of mathematicians and astronomers and physicists; students of social phenomena, biologists, and educators planning a new science of education; from these statistics has descended.[1]

Today the field of statistics has many followers, yet there are many more who feel that any one who deals with numbers, and hence statistics, cannot be trusted. Even though there is a strong negative view toward the concepts of statistics, it has nevertheless become firmly established as an indispensable tool of the comptroller and scientific management.

Statistics has become a basic tool of decision making and is a basis for many important management functions such as market and scientific research, budgeting and forecasting,

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[1]Helen M. Walker, Studies in the History of Statistical Method (2d printing; Baltimore, Md.: The Williams and Wilkins Company, January 1931), p. 1.

purchasing, production planning, quality control, engineering, etc. Of these broad applications the military comptroller is vitally interested in reliable methods and techniques for budget prediction and forecasting.

Many in the field of comptrollership do not understand how this vital tool can be used to their advantage; there is inadequate understanding of this powerful tool even among many persons engaged in the fields where statistics has its greatest application.

One might ask why this new technique of scientific management is not better understood and accepted. Some educators give as the paramount reason the fact that statistics was a rare course in our colleges during the college years of most of our key executives today. And even today one can still find many schools of business administration of some of our leading colleges that do not have an adequate statistical curriculum.

What Is Statistics and Why the Growing Interest?

Most educators will define statistics as that branch of applied mathematics which deals with the collection, tabulation, presenting, analyzing, and interpreting quantitative data.

> There are many reasons for the growing interest in the analysis of quantitative data. One is that the tools of analysis have become sharper as refinements have been made in statistical techniques. But the principal reasons for the increasing use of these methods are to be found in certain basic changes which have taken place in the American economy.
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> First, there has been a rapid transition from the day of small local enterprise to an era of centralized large-scale production. As the size of the business

unit has increased, the importance of executive decisions has increased. Today, great corporations exceed in wealth the sovereign states which created them. Never before have business executives controlled so much wealth, nor have so many persons been dependent on a single enterprise, and never before have mistakes been so costly in terms of money and human suffering.

The responsibilities of the modern executive have grown with the scale of his operations, but his ability, as an individual, to reach proper decisions has not kept pace. A. C. Nielsen, of the A. C. Nielsen Company of Chicago, is a marketing expert who employs quantitative data in analyzing problems of marketing for some of America's largest corporations. Always, before submitting an analysis of a particular problem to the executive who has ordered it, he asks the executive what he considers the answer to be. He has kept a record of the replies. According to Mr. Nielsen the executives have been wrong 42 per cent of the time.

Lacking objective statistical studies to guide them, the executives would have made almost as many wrong as right decisions.

A second development in the American economy which has stimulated the use of quantitative data is the great variation which has appeared in economic activity as cyclical prosperities and depressions have followed one another in rapid sequence. One effect of the apparently increasing instability of the economy has been to create an urgent need in the fields of business and government for a type of analysis of economic conditions which may enable the executive to see a short distance into the future, to detect the shape of things to come shortly, at least, before they arrive with the devastating force of depression or the buoyant effect of prosperity.[2]

There has been a growing use of statistics in government as well as in business. As the government continued to grow in size and to assume many functions statistical techniques were increasingly emphasized. The complexities of planning for the economy as a whole, the obligations assumed by the various federal bureaus in administering the affairs of

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[2]William Addison Neiswanger, Elementary Statistical Methods (New York: Macmillan Company, 1951), p. 2.

industry and agriculture, and providing for the national security means that decisions made by government officials are of great importance in terms of money and human welfare. In this area it is obvious, mistakes can be most serious, and decisions based on personal direct observation are unlikely.

The duties and responsibilities of the comptroller of the Navy stem directly from those specified in Title IV of the National Security Act Amendments of 1949. Title IV states among other things that the comptroller shall:

> Establish and supervise principles, policies, and procedures of organizational and administrative matters relating to the preparation of cost budgets, fiscal operation, capital property, accounting, and progress and *statistical reporting*. (italics mine.)[3]

The Secretary of the Navy in setting forth the basic concepts of comptrollership in the Navy emphasized the statistical function by requiring the comptroller to be responsible for "developing guides and criteria for the *collection and coordination of statistical data and preparing special statistics as required*." (italics mine.)[4]

The Controllers Institute's Committee on Ethics and Eligibility Standards emphasizes statistics as a Comptrollership function and states among other things that the controller is responsible for the compilation of statistical records as required.

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[3]U. S., Congress, *National Security Act Amendments of 1949*, Public Law 216, 81st Congress, 1st session, 1949, Title IV.

[4]U. S., Department of the Navy, *Secretary of the Navy Instruction 5400.4*, November 18, 1953, p. 1.

At this point it might be well to emphasize this point that it is not the purpose of this paper to imply that the comptroller be a trained and accomplished statistician. The comptroller needs to make decisions rapidly, based upon timely and accurate data. It is only through the modern methods of statistics and applied mathematics that such a desire for data collection can be realized; thus it becomes important that the military comptroller while essentially a generalist and not a specialist, at least among line officers, become thoroughly familiar with the capabilities and relationships of statisticians and statistics in the broad field of management.

Management is concerned with what must be done, thus it becomes necessary for the manager to make some sort of forecast of what events are likely to happen. Often this is accomplished by using past experience as a guide, or his intuition but, although these can at times be good guides, they can also be disastrous mistakes which could have been avoided were more accurate and timely data available and effectively used.

To be effective management needs to know such things as the relationship existing between one set of events and another, and the trends established by these events, and causes of these events in order to add a degree of reliability to its forecasts. Usually no invariable relationships of cause and effect can be found in the past or predicted of the future and so the manager has to deal in the realm of probabilities. This he is by training used to doing as a result of mostly dealing

with human relationships and individual persons, whose individual actions cannot be predicted with any degree of certainty.

There are still those in the field of management, however, who are afraid of figures. They feel that they can see and hear what is going on and do not need figures to tell them. But it is impossible to be everywhere at the same time and reliance must be placed, on occasion, on the observation and judgment of others--within these observations is necessarily the element of measurement and measurement cannot be possible without the use of numbers.

The techniques of drawing conclusions from numbers is where misunderstanding and harm are done. Intuition can provide for better conclusions than those of the man who loves figures, but does not have the special knowledge of how to deal with them, and works out percentages and averages and then draws unjustifiable conclusions from them.[5]

> It is still too little realized that there exists a technique of dealing with figures in such a way as to find out with a very high degree of probability what sort of conclusions, if any, can validly be drawn from certain sets of figures with what degree of accuracy they can be drawn and to present those conclusions in a simple form. In this way not only is it possible to avoid drawing wholly unwarranted conclusions, but also at time to make some sense of a set of figures which at first sight may appear to tell nothing.[6]

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[5] The Report of a Committee, for Submission to the Twenty-Fifth Anniversary Conference of Management Research Groups at Buxton, 1-4 June, 1951 (London: Management Research Groups, Manfield House), p. 6.

[6] Ibid.

These techniques of dealing with figures is the service that the trained statistician specializing in mathematical analysis can give to the comptroller or manager, and in subsequent chapters of this paper examples of these techniques will be presented.

> The basis of all statistics is counting or measurement and the statistician specializes in methods of counting or measurement, whether by so-called "complete counts," or censuses, or by counts or measurements of a sample of the whole, from which the totality can be estimated. To this end he uses a special set of applied mathematical techniques based on the theory of probability.
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> Statistical studies are of two classes, enumerative and analytical. Enumerative study determines how many of some classes of things exist at a particular time, but is not concerned with why they exist. An analytical study investigates patterns and variations either in one series of events over a period of time, or between different groups of things. It may or may not be possible as a result to make some assessment of causes, but if the results of the study are at all positive then some causal relationship is assumed to exist. The routine calculation of enumerative data often forms the basis for analytical studies.[7]

It is important at this point that the statistician and the accountant not be confused--that a distinction be made between the two arts. In accounting the individual account or entry is of chief concern. It must be accurate and reliable and subject to audit. In statistics it is not necessary to have accounting accuracy for each individual item, but rather estimates based on sampling is the order of the day. Mathematical accuracy is still required in statistical work and it is not intended to imply otherwise, however, some sacrifice of reliability can be made of the complete count for a sampling study expertly done.

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[7] Ibid.

The unit loses its identity in a statistical measure such as an average or a total. It is the measure or total and the picture portrayed by these measures from which decisions are based and not from each individual unit. The statistician may say that a certain fact exists within a specified range such as the inventory of a specific item is not more than 1050 items, for example, nor less than 950 items. The accountant would be required to produce a complete count capable of audit.

Conclusion

In conclusion it is well at this point to summarize this discussion and to point out some valid reasons why the comptroller and management should acquire statistical knowledge, and the relationship between management, the comptroller and the statistician.

I think it is generally understood that the comptroller is to operate in a staff capacity, however, he may recommend to management, but he does not make managements decisions. Management looks upon the comptroller as a continuing source of ideas and advice on managerial problems. The comptroller must have had broad experience with operating programs and problems and general management responsibilities. He should have a keen analytical ability and be capable of making discriminating judgments.

Since statistical methods are capable of application to all of the activities of an organization, and are a service function it is fitting then that this function should come

under control of the comptroller or financial director, and this it does in most cases.

There are many valid reasons for management to understand statistics. Such an understanding of course would promote a wider use of statistical data in management since problems where statistics can be profitable would be quickly recognized.

Communication with the professional statistician would be improved by a better understanding of the art by management. Albert Einstein once said, "The formulation of the problem is often more essential than the solution." With a statistical background management can understand the problems involved in fact-finding and thus can avoid misunderstanding in research projects conducted in the organization and assure effective use of the complied data.

Management must understand that the science of statistics is not infallible and that there is risk involved in decision making based on statistical conclusions. Most statistical decisions are based on samples and this involves risk. A statistical minded management should understand the "risk" involved and the "confidence" in statistical decision making.

Of course a better understanding of statistical methods serves to emphasize that good effective statistical studies require a trained staff of professional statisticians. This will yield valid results with a minimum of cost. Statistical functions cannot be assigned to the budget analyst,

clerk or bookkeeper in the office or the foreman or technician in the plant.

And finally management will become a more confident administrator and enjoy peace of mind, insofar as administrative matters are concerned, by knowing the basic elements of statistics.

More important he will make less mistakes and will be better able to control the mistakes of others in his organization.

CHAPTER II

HISTORY OF STATISTICAL METHOD

The Normal Curve

Origin in the Theory of Probability

The history and development of statistical method is a colorful and interesting study. The statistical methods of today are based on the science of probability. The early scientists and mathematicians were aware that certain natural activities occur in conformity. The early observations of Aristotle and astronomers in establishing fundamental physical laws of the universe, regimented certain observations in the field of science. The ability to predict happenings and behavior on the basis of new found laws provided the authoritative position of charlatans, mystics, magicians and gamblers. Of course at this stage of development it was impossible to predict the behavior of certain factors which were caused by chance alone, however, this did not discourage or weaken the position of these early philosophers and scientists. The real beginning of the science of probability did not occur until the early part of the 17th century.[1] The exact date for the

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[1]Robert Kirk Mueller, Effective Management Through Probability Controls (New York: Funk and Wagnalls, 1950), p. 62.

discovery of the normal curve (fig. 1 ch. III) which had its beginning and discovery from the theory of probability was November 12, 1733.

There were some faint traces, of no more than six or eight problems, that intimated the idea of probability before the year 1600. These traces appeared in the Orient. One example occurred when the Chinese writer Yüan Yúan criticized a still older writer, Sun-Tze for the following reason: "Many unnecessary details appear in his works on mathematics, such as a certain absurd problem, which surely cannot be attributed to him, on the probability that an expected child will turn out to be a boy or to be a girl."[2]

In Europe the really first references to the theory of probability is a statement in a commentary (Venice, 1477) on Dante's Divine Comedy concerning the different throws which can be made with three dice. The first writer to discuss a problem concerning gambling in a mathematical paper was Luca Pacioli in his Suma (1494). In this paper he gave the first version of the problem concerning the equitable division of the stakes between two players of unequal skill when the game is interrupted before its conclusion. This problem was repeated and amplified in all the works on probability for two hundred years. Astronomers played a conspicuous role in developing the mathematical theory of probability two centuries later--it is worthy of note that both Kepler and Galileo made brief

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[2]Helen M. Walker, Studies in the History of Statistical Method (Baltimore, Md.: The Williams and Wilkins Company, January 1931), p. 1, ff.

reference to the subject of chance.

The first real scientific works on probability began in the 17th century. The foundations were laid by the two most distinguished mathematicians of that time, Blaise Pascal and Pierre de Fermat. The theory developed when the Chevalier de Méré, a gambler having unusual ability, even for mathematics, proposed certain questions to Pascal. Among these questions was the celebrated "Problem of Points" concerning the division of stakes between two players who separate without completing their game. Pascal and Fermat exchanged numerous letters on this subject during the year 1654, and in the course of this correspondence they generalized the problem more and more until at its close, that which had first been a source of perplexity to a gambler had been elevated to an important mathematical concept.

Within the next few years several problems relative to the outcome of games of chance were proposed and solved in the journals of various learned societies by Jacques Bernaulli, Montmort, De Moivre, Arbuthnot, Francis Roberts and others.

The Discovery of the Normal Curve

The normal curve[3] was discovered by Abraham De Moivre (1667-1754) as a result of his intensive study to his work and

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[3]The Normal Curve: A symmetrical bell shaped-curve possessing certain statistical characteristics. See discussion Chapter III.

his applications of the binomial theorem[4] led him to discover a formula for the ratio between the middle term and the sum of all the terms of $(1+1)^n$. Many of the recent treatises on probability and sampling approach the matter by a method similar to De Moivre's use of the binomial expansion.[5]

This formula was first published November 12, 1733. In 1730, De Moivre introduced his Miscellanea Analytica and three years later he presented privately to a few friends a short seven page paper entitled Approximatis ad Summan Terminorum Binomie $a+b^n$ in Seriem Expansi. In this obscure treatise on abstract mathematics, written in Latin, two centuries ago, and believed by De Moivre to have no practical implications outside the realm of games of chance, we have the first formulation of the concept of the law of errors. He did not realize that there would come a time when this theorem would powerfully affect the thinking of the world on all its social problems, shape the policy of educators in the schools, and aid in thousands of investigations in science whose very names were then unknown.

Later De Moivre announced the formula for the curve of error, and connected his theory of probability with theology,

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[4]Binomial Theorem: The theorem by means of which a binomial may be raised to any power without performing the multiplications. A binomial is an expression consisting of two terms connected by a plus or a minus sign. A binomial distribution is a distribution where there are only two alternatives. For example in sampling light bulbs, they are either good or bad.

[5]Binomial Expansion: Raising the binomial to a given power.

urging that this tendency of events to conform to law argues a Great First Cause.

> And thus in all cases it will be found, that although chance produces irregularities, still the odds will be infinitely great, that in the process of time, those irregularities will bear no proportion to the recurrency of that order which naturally results from Original Design. . . . Again, as it is thus demonstrable that there are, in the constitution of things, certain laws according to which events happen, it is no less evident from observation, that these laws serve to wise, useful and beneficient purposes, to preserve the stedfast Order of the Universe, to propagate the several Species of Beings, and furnish to the sentient kind such degrees of happiness as are suited to their state.[6]

In these later writings, De Moivre is eager to free the subject of probability from its connection with gambling, and establish a theological doctrine of a divine order working through human affairs and exhibiting itself in the regularity of statistical ratios.

One of the first attempts to reduce the theory of probability to rules of thumb which might be followed by persons having no command of mathematics beyond simple arithmetic was made by Augustus De Morgan (1806-1871) in An Essay on Probabilities and on Their Application to Life Contingencies and Insurance Offices. De Morgan was brilliant and somewhat erratic; he wrote on a wide range of mathematical topics. He was the greatest English authority of his day on the theory of probability. He stated in his Essay on Probabilities that heretofore the work in probabilities has been possible only to the expert mathematician, but that

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[6] *Ibid.*, p. 17, ff.

> those who already admit that the theory of probabilities is a desirable study, must of course allow that persons who cannot pay much attention to mathematics, are benefited by the possession of rules which will enable them to obtain at least the results of complicated problems, and which will therefore, permit them to extend their inquiries further than a few simple cases connected with gambling.

His book was really a handbook, full of practical advice and sufficient practical rules for the treatment of a set of observations. He advised against unwarranted conclusions from a situation.

Later De Morgan wrote other essays on the Theory of Probability and in these he discussed the theory of least squares,[7] the application of the calculus of definite integrals, and some complicated problems in probability. One portion of his essay is of particular interest because it indicates the direction and thinking of scientific men of that day. This particular section discusses the probabilities of testimony, of miracles, of moral questions, of decisions of a jury. He went so far as to suggest that an investigation into the validity of the English practice of requiring a unanimous decision from a jury might be made by studying five hundred trials in which the jury have delivered their verdict at once to see if they show a smaller percentage of error than five

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[7]Mathematically computed lines may be passed through the data, or scatter points, figures 2, 3, 4, 5, chapter III which are called regression lines, or lines of average relationship, because they reveal the typical change in the dependent variable Y which has, in the past, accompanied a given change in the variable plotted on the X axis. This average relationship may be determined mathematically by the method of <u>least squares</u>.

hundred trials in which the jury deliberated for two hours or more. Other writers later attempted to assign numerical values to the probabilities of the accuracy and sincerity of witnesses, and by elaborate mathematical processes attempted to reach numerical measures of the probability that a given defendant is guilty or innocent.

Moments

Professor Karl Pearson is generally regarded as having accomplished the most significant work in the development of moments. The analogy between the statistical function obtained by taking the sum of the nth powers of the deviations in any given distribution, and the mechanical concept of a moment, had been noticed by other writers before Pearson, but none of them had perceived in that function the powerful tool which he developed.[8]

Pearson studied extensively the effect of sampling upon the moments of a frequency distribution. Pearson and his associates set down the general method by which the sampling error of any moment might be studied. Among the formulas derived were for the standard deviation,[9] the standard deviation of a coefficient of correlation,[10] the correlation between

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[8] *Ibid.*, p. 76, ff.

[9] Standard deviation--the most reliable measure of variation. Often referred to as the root-mean-square.

[10] Correlation coefficient--an index measuring the extent of relationship between two variables. It ranges from -1 (a perfect negative relationship) to 1 (a perfect positive relationship).

sampling errors in the standard deviations of two correlated traits, and the correlation between sampling errors in a coefficient of correlation and in a standard deviation. The moments are used in curve fitting and from the moments are derived the parameters[11] in the Pearson system of curves.

Percentiles

The discovery of the importance of the median as a measure of central tendency was made about seventy five years ago. There were, however, some earlier writers who had conceived this measure but who did not place great emphasis upon it.[12]

In 1816 Gauss suggested a simple method of computing the probable error. He said "let all the errors be arranged in order of size without regard to sign, and take the middle one if the number is odd, or the mean of the two middlemost if the number is even." This value, which he called merely M, is thus the median of the absolute value of the errors, and similar to, though not necessarily identical with Galton's quartile deviation.

Encke, Quetelet, Fechner, and Galton made similar studies. Sir Francis Galton's, (1822-1911), first statistical work was a study of Hereditary Genius (1869) which led him to feel the need for a satisfactory method of reporting differences in achievement and intellectual ability. The following are

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[11] Parameter--a characteristic in a population. For example, the true proportion of defectives in a lot of 50,000 light bulbs.

[12] *Ibid.*, p. 83, ff.

some quotes from his book:

> The theory of hereditary genius, though usually scouted, has been advocated by a few writers in the past as well as in modern times. But I may claim to be the first to treat the subject in a statistical manner, to arrive at numerical results, and to introduce the "law of deviation from the average" into discussions on heredity. . . . The range of mental powers between . . . the greatest and least of English intellects, is enormous. There is a continuity of natural ability reaching from one knows not what height, and descending to one can hardly say what depth. . . . I propose in this chapter to range men according to their natural abilities, putting them into classes separated by equal degrees of merit, and to show the relative number of individuals included in the several classes. . . . The method I shall employ for discovering all this, is an application of the very curious theoretical law of "deviation from and average." First I will explain the law and then I will show that the productions of natural intellectual gifts come justly within its scope. . . . Here we arrive at the undesirable, but unexpected conclusion, that eminently gifted men are raised as much above mediocrity as idiots are depressed below it; a fact that is calculated to considerably enlarge our ideas of the enormous differences of intellectual gifts between man and man.[13]

The idea of grades has its first published statement in this book Galton placed the ablest men in each million in the highest grade, and the most stupid in the lowest grade, and then divided the remaining 99,998 into 14 classes, the average ability of each being separated from that of its neighbors by equal grades, thus forming a table to be applied to special or general ability. It would be true for any type of examination whether held in painting, in music, or in statesmanship. The proportion between the different classes would be identical in all these cases, although the classes are made up of different individuals. It is clear that Galton was moving rapidly in the direction of a percentile scale, although even the concept

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[13] _Ibid_., p. 87, ff.

of the median is not explicit as yet.

Galton later developed percentile scales in assigning marks of excellence on test scores he arranged all the members of the group in an order of merit. One of Galton's earliest attempts to do this scientifically resulted in a "scale of merit" for the men who obtain mathematical honors at Cambridge.

Galton's general method of grades and deviates is widely used in assigning class marks, and in constructing educational tests and scales. It is fundamentally the method employed for the construction of a T-scale.[14]

Correlation

It would be rather difficult to condense a complete history of correlation in this paper. It is the purpose here to present only a brief account of the development of correlation.

Correlation theory developed historically from the theory of probability, and it is generally understood that correlation was the unique discovery of Sir Francis Galton, made early in the last quarter of the nineteenth century, and greatly elaborated and refined by Pearson, Edgeworth, and Weldon. Many mathematicians came close to the discovery of correlation. Several French and German astronomers, physicists and mathematicians approached the problem from the point of view of mathematical analysis, but failed to see the practical

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[14]A theoretical distribution that goes by the name of Student-T distribution, developed by W. S. Gosset who published his work under the pen name of Student.

significance of the formulas they derived. It has been said that this may be as a result of too much attention to theory and not enough toward empirical data.[15]

In his Memories, Galton tells of the first time he recognized that regression would be the key to the problem of heredity (the concept of correlation developed from regression). Galton discovered that the laws of heredity were solely concerned with deviations expressed in statistical units.

Galton's earliest studies on regression were in 1875 when he was conducting experiments with sweet-pea seeds to determine the law of inheritance of size. The question at this time in Galton's mind was: "How is it possible for a whole population to remain alike in its features, as a whole, during many successive generation if the average produce of each couple resemble their parents?" From the crude data obtained and assembled the law of regression; the formula for the standard error of the estimate,[16] and the idea that the various arrays in the correlation table had equal variability.

> In studying the inheritance of stature, Galton first transmuted all female statures to male, multiplying them by the constant 1.08 (this had the effect of equating means, but did not take into account the different variability in male and female statures). The average of the statures of the parents was then

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[15] Ibid., p. 105, ff.

[16] Standard error of estimate--a measure of the scatter about the line of regression. The standard deviation of the observed points about the line.

> termed the "mid-parent;" the average of the statures of the offspring, the "mid-Fraternity." From these he made "Tables of Stature," later called correlation tables. For each array and for the table as a whole he found the median. He also found Q (semi-interquartile range)[17] used as an approximate value of the probable error, for the general population, for the mid parents, and for each "co-Fraternity," a "co-Fraternity" being all of the adult sons and transmuted daughters of a group of Mid-Parents who have the same stature (reckoned to the nearest inch). Examining these values of Q for the various Co-Fraternities, he then found them roughly the same, regardless of whether the mid-parents were tall or short. Examining the medians he found that "However paradoxical it may appear at first sight, it is theoretically a necessary fact, and one that is clearly confirmed by observation, that the Stature of the adult offspring must on the whole be more mediocre than the statures of their parents." As the medians of the rows and columns proved to lie on approximately straight lines, he called the latter, lines of regression.[18]

Because Galton used the variability in his case Q as the unit in which he expressed deviations, these lines of regression were symmetrically placed. In 1877 he called the slope of these lines r for <u>reversion</u>. In his paper "Regression Towards Mediocrity in Hereditary Stature" (1886) he used W, and M "Correlations and their Measurement" (1888) he returned to r, which now stood for regression.

Galton defines correlation as follows.

> Two variable organs are said to be co-related when the variation of one is accompanied on the average by more or less variation the other, and in the same direction. . . . The statures of kinsman are co-related variables; thus the stature of the father is correlated to that of the adult son, and the stature of the adult son to that of the father; the stature of the uncle to that of the adult nephew, and the stature

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[17] Inter-quartile range--the middle 50% of the observations.

[18] <u>Ibid</u>., p. 106, ff.

of the adult nephew to that of the uncle, and so on.[19]

Operations Research

To give a complete account of the development of the technique of operations research would require several volumes, far more than the few paragraphs that are allotted to the subject in this paper. Ever since about the third century B. C. political and military leaders have consulted scientists for solutions to the problems of war. There are many instances on record where these non military specialists have been able to assist the military with new ideas about the machines, tactics and, strategies of war.

Operations Research began to produce more and more case histories about the time of World War I as both England and the United States attempted to analyze military operations by using mathematical methods. F. W. Lanchester of England produced some papers in 1914 on the relationships between victory, numerical superiority and the superiority of fire power. In America Thomas Edison made studies of antisubmarine warfare for the Naval consulting board which included the compilation of statistics to be used in determining the best methods for evading and for destroying submarines.[20]

By 1939 at the outbreak of World War II in Europe there was the nucleus of a British operational research organization

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[19] *Ibid.*, p. 106.

[20] Joseph F. McCloskey and Florence N. Trefethen (ed.), *Operations Research for Management* (Baltimore, Md.: John Hopkins Press, 1954), p. 4, ff.

already in existence. Some of the problems under study at this time were to assist military personnel with radiolocation and the integrating of the newly developing radar system of early warning against enemy air attack with the older system of operational control based principally on the observer corps, whose members were trained in the sighting, identification, and reporting of planes. Other phases of operations research consisted of an analysis of all phases of German air activity during night operations over Britain; problems concerning the detection of ships and submarines by the use of radar equipment in airplanes, and all phases of antisubmarine warfare.

Studies of the data obtained from the systematic bombing raids by the Allies over Germany indicated that larger plane formations suffered smaller percentage losses. These findings resulted in the first 1000-plane R. A. F. raid over Germany. In the United States the Army Air Force and the Navy began work in the field of operations research in 1940 after Dr. James B. Connant, then chairman of the National Defense Research Committee, visited England and became aware of the techniques of operations research. As in Britain early operations research for the Air Force grew up around problems arising from new radar equipment. Toward the end of the war in the Pacific, the U. S. Navy's operational research team undertook a crash project in operations research as a result of the onslaught of kamikaze attack against Allied ships. The question to be answered was: "Should the ship under attack maneuver violently to avoid being hit or keep straight in order to get

better aim with its anti-aircraft guns?" The operations research team working on the problem obtained the records of 477 attacks, including 172 hits and 27 sinkings. The conclusions reached after analysis was that a large ship should maneuver violently, whereas a small ship should change course slowly. Recommendations were also made about how ships should turn to receive inevitable hits. Tabulations of succeeding attacks demonstrated the validity of the recommendations. Those ships under attack which observed the recommendations were hit 29 per cent of the time, whereas others were hit 47 per cent of the time.

Nonmilitary operations research is probably best exemplified by the work of Horace C. Levinson, one of the first proponents of operations research in the United States. He began his work in the 1920's after giving up a career in astronomy for another in merchandising where he applied the trained scientist's methods to the problems of his employer. His work included studies of customers' buying habits, the effect of quick response advertising on sales, and the relation of neighborhood environment to types of articles sold. There were many others. The field of management consulting has much in common with operations research and had its beginning in the latter part of the 19th century, when such pioneers of scientific management as Taylor, Gantt and Emerson started their work.

The study of operations research eventually found its way into the Universities. In 1948, the Massachusetts

Institute of Technology established, in collaboration with the Navy a course in non military applications of operations research. Similar courses were established at the University College, London in 1949 and Birmingham University in 1950. The Case Institute of Technology became the first institution of higher learning to offer a curriculum in operations research leading to a degree of Master of Science. Later Columbia and John Hopkins followed with courses in operations research.

Today operations research appears to be in an intermediate stage of development. It has proven its usefulness by a history of many cases, however, it is still developing with such speed that even the most experienced among operations research workers hesitate to mark out all the problem areas that are not susceptible to an operations research approach.

CHAPTER III

STATISTICAL METHODS AND CASE STUDIES

Although most business executives have a reasonable background of mathematical training, the secrecy, mystery and terminology with which the statistician and technical man surround the subject of statistics are often as confusing as the White Queen's example of addition.

> "Can you do addition?" the White Queen asked (of Alice). "What's one and one and one and one and one and one and one and one and one and one?"
> "I don't know," said Alice. "I lost count."
> "She can't do addition," the Red Queen interrupted. . ..
> <u>Alice in Wonderland</u> by Lewis Carroll

You need not be a mathematician to understand the basic principles used by statisticians or to use a statistical approach to problems in comptrollership and business problems in general; in fact addition, subtraction, long division, and an occasional square root thrown in is about all that is needed for an executive to become reasonably familiar with statistical techniques.[1]

It is impossible within the scope of this paper to present in detail the many statistical methods used by statisticians in obtaining solutions to the many different

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[1]Robert Kirk Mueller, <u>Effective Management Through Probability Controls</u> (New York: Funk and Wagnalls, 1950), p. 74, ff.

types of problems; nor is it possible to attempt to make trained statisticians out of those who read this paper, but, rather it is intended, that this chapter shall be a non technical discussion of some of the more important and useful statistical techniques used in decision making, and the presentation of the highlights of some actual problems and the method used in their solution.

It is assumed that the reader has had some training in statistics and is familiar with the measures of central tendency (mean, median and mode), frequency distributions, measures of variation (the range, inter quartile range, mean deviation) and the most important of all measures of variation the standard deviation, and correlation and regression. If this is not the case I refer you to the text books listed in the bibliography and to Appendix B of this paper which contains a sample problem using simplified numbers to demonstrate the method of obtaining some of these various measures. Correlation and regression is taken up in detail in the latter part of this chapter.

Probability

To provide a proper background for the discussion of statistical method it is well to start with the theory of probability since statistical methods are based on this theory. The theory of probability is aptly described by the French mathematician Emile Borel as follows:

> The theory of probability is of interest to artillerymen; it is of interest, also, not only to card and dice players, who are its godfathers, but

> to all men of action, heads of industries or heads of Armies, whose success depends on decisions which in turn depend on two sorts of factors, the one known or calculable, the other uncertain and problematic; it is of interest to the statistician, the sociologist, the philosopher.[2]

In lay language probability may be defined in its subjective sense as a state of mind as to the degree of certainty or uncertainty. A second interpretation is that which usually happens and may be defined as the relative frequency with which an event occurs in a certain class of events. The third interpretation of probability holds that probability refers not to events but to propositions and has nothing to do with intensity or belief or the statistical frequency.

> All human affairs rest upon probabilities, and the same thing is true everywhere. If man were immortal, he could be perfectly sure of seeing the day when everything in which he had trusted should betray his trust and in short of coming eventually to hopeless misery he would break down at last as every good fortune, as every dynasty, as every civilization does. In place of this we have death. . . .[3]

With improved accuracy of measurement, it has been determined that no two things are exactly alike. In the case of a manufacturer of uniform items there exists a family of chance causes of variations, such as tool wear, fluctuation in voltage, variations in operator performance, etc. When all of the controllable factors are in a state of control, these chance causes which are not controlled vary according to a definite symmetrical pattern known as the <u>normal frequency distribution</u>

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[2] Ibid., p. 62.

[3] *Ibid.*, p. 67.

curve, figure 1.[4]

The abscissa or the peak of the curve represents the central tendency or average of all measurements with the number of observations or measurements varying in accord with the curve to the left and right of the center line. It can be noted that the number of observations or measurements which are farthest from the average are very few in number, but they still exist.

> Once the mathematicians had determined that a system or process undisturbed and subject only to chance variations, could be arrayed in a frequency distribution curve this information was put to work practically by further developing a mathematical theory about the characteristics of the points "scattered" about the average or central tendency of the diagram. The "scatter" points are independent of the average central tendency. A characteristic of the frequency distribution curve will be seen in following the curve from the left-hand side upward where it is noticed that the direction of this curve changes from concave to convex and passes through a flat point a little over half way towards the peak. Following the curve up over the peak and back down it is seen that this tendency is changed to where the curve goes from convexity to concavity after passing through another flat point. The flat point can be determined geometrically and is a characteristic of each ideal frequency distribution. The distance between this flat point and the mean is the same on either side of the central tendency line and has been arbitrarily called a "standard deviation" designated in mathematical terminology by the Greek letter sigma (σ).[5]
>
> It has been determined that the number of parts occurring in the area bounded by sigma on each side of the middle line corresponds to 68.3% in an ideal frequency curve. It has also been determined that the number of parts or measurements which occur within two sigma limits, that is, twice the standard deviation distance on either side of the central average, amounts to 95.5 percent of the occurances or parts. The three-sigma limit reaches

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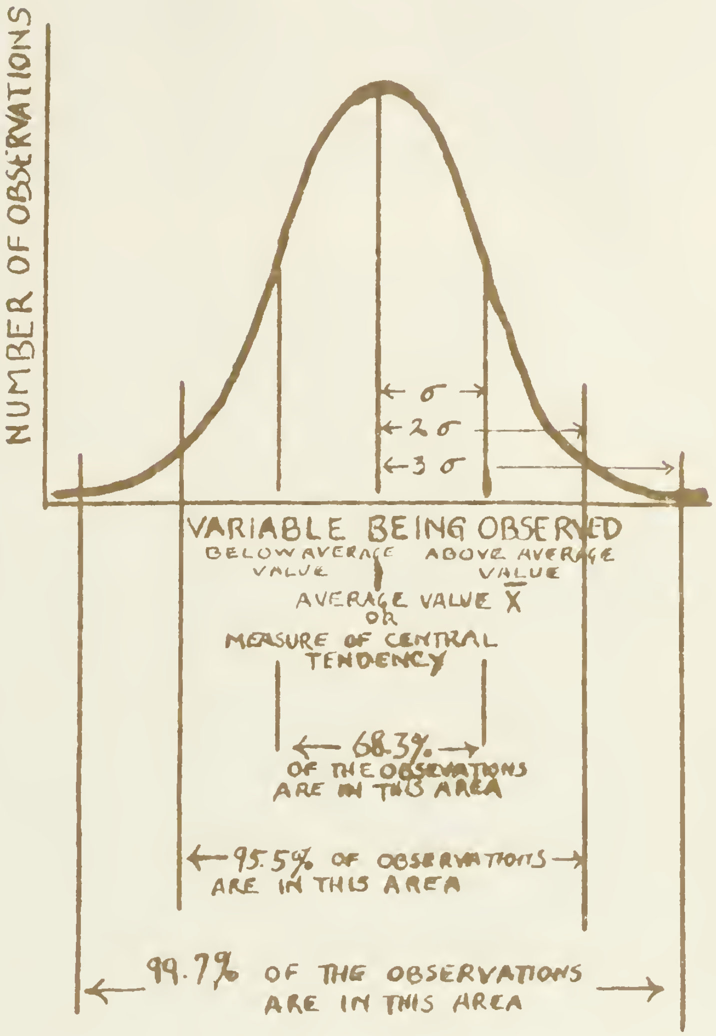
[4]Ibid.

[5]Ibid., p. 69.

NORMAL CURVE



MATEMATICAL EQUATIONS HAVE BEEN DEVISED TO DEFINE THIS CURVE AND VARIATIONS FROM IT. THIS BELL SHAPE CURVE IS ALSO REFERRED TO AS:

THE PROBABILITY CURVE
THE NORMAL CURVE OF ERROR
THE GAUSSIAN CURVE
THE LAPLACIAN CURVE
THE NORMAL DISTRIBUTION CURVE
THE NORMAL LAW

FIGURE 1. IDEAL FREQUENCY DISTRIBUTION CURVE

> out into the asymptotic zone and accounts for 99.73 percent of the parts, and this three-sigma limit is often adopted as an economical and practical basis or limit for commercial use of statistical control. These limits are such that an ideal process operating under the influence of chance variability will produce all production items within these percentage limits. In mathematical language this is referred to as "plus or minus three standard deviations" or "three sigma limit."[6]

In most cases of industrial or scientific decision making it is generally accepted that a probability of 95 per cent (two sigma) is the minimum limit for a good decision. To hold out for 99 per cent confidence would in most cases involve considerable expense. When working at the 95% confidence level, it is most probable that a correct decision is made at least 19 times out of 20.

Sampling

Everyone who has considered business problems knows that data on many situations are sometimes required under circumstances where it is not economically feasible or even possible to take measurements, enumerate, or test the entire universe or population under consideration. Very often, however, a sample may be of great assistance, and more often than not a sample will provide more accurate results than the complete data. Although a complete enumeration will avoid making sampling errors, overall accuracy can be increased by conducting a sample survey rather than a complete count. For example the smaller scope of the sample project makes it possible to be more selective in assignment of personnel for

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[6] *Ibid.*, p. 70, ff.

the survey operation; to be more thorough in their training; and to be able to concentrate to a much greater degree on reduction of sampling errors. The net effect of a sample survey, as compared to the 100% count, is often in addition to being more accurate, is achieved with fewer personnel, less paper work, at a lower cost, and in a shorter time.[8]

It is well at this point to distinguish between a population (or universe) and a sample. A population is the entire set or group about which we desire some type of information, while a sample is a subset or a part of the population. In the population there are certain characteristics which we may estimate or which we may desire to observe and control. These characteristics are called parameters. The corresponding characteristics in a sample are called statistics.

Sampling is the technique of drawing a few items from a universe or mass in such a way as to secure statistics on the basis of which we may, by inference, construct a picture of the mass from which the items come. The technique of sampling is used because frequently it is the only possible, or practical, and usually the most efficient means of studying mass data. A situation in which sampling is the only possible method would occur when materials are tested for quality, the object may be destroyed as would be the case in testing bricks or steel parts, for example, for tensil strength or for resistence

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[8] M. J. Slonim, "Sampling in a Nutshell." Directorate of Statistical Services, DCS/ Comptroller, Headquarters, U.S.A.F., p. 16, ff.

to crushing forces. Sampling would be the only practical method for example when the statistical universe is large and composed of thousands or even millions of items. To enumerate the entire population would be impractical from a time and cost point of view. Even though in some case it would be possible from a physical or financial standpoint to make a complete count, as was pointed out above, it is not necessary to do so. A sample that is properly taken will provide the necessary data from which generalizations about the population can be made, and far more efficiently. Briefly let us examine a few of the sampling techniques that can be used for any of the problems the statistically minded comptroller may encounter.

Simple Random Sampling

Simple random sampling is simply a sampling technique in which chance alone dictates which elements of the population will be members of the sample. All subjectiveness and human bias have supposedly been removed. In a random sample each element of the universe or population has an equal chance of being selected. If, under these conditions a sufficiently large number of items is collected, the sample will be a miniature cross section of the mass from which the items come. Random samples are not haphazard selections, but rather the system of selection is controlled and is not a hit or miss selection.[9] Tables of random digits have been prepared to assist in obtaining a truly random sample from a population.

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[9] William Addison Neiswanger, Elementary Statistical Methods (New York: Macmillan Company, 1951), p. 96.

Stratified Sampling

There are particular situations when it is desirable to have all elements of the population represent. For example, when making a study of inventory items, a simple random sample may exclude the more costly items at the extreme range of the population from being selected when it is desirable or necessary to have some information about these costly items. Stratified sampling is a scheme for accomplishing this task while at the same time retaining the desirable property of a random sample.

To accomplish this the population is divided into homogeneous groups, referred to as strata--each element in the group is similar to the other elements within the group. For example in the case of inventory items we may establish groups or strata of (1) all items valued at less than $5.00 (2) items valued from $5.00 to $25.00 (3) items valued from $25.00 to $99.00 and (4) all items that exceed $100.00. Now that the strata or groups have been selected or established we sample randomly from within each strata.[10]

Cluster Sampling

There are on occasion situations in which the ease of sampling and the cost are important, and we are unable to select a simple random or stratified sample from a given population. In this situation we may divide the population

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[10]Chester H. McCall, "A Statistics Manual." Unpublished statistics manual prepared for the Navy Graduate Comptrollership Program, Department of Statistics, The George Washington University, Washington, D. C., 1958, p. 34, ff.

into clusters which are, in fact, miniatures populations, and then to select at random, several clusters, or a single cluster, from the population of clusters. From these clusters we may then draw inferences about the whole population. In order for this technique to be successfully applied the clusters must be similar to one another in their characteristics. For example, if we assume that all destroyers should have crews with similar attitudesregarding retirement and fringe benefits, then the selection of a single crew, or several crews would suffice to draw some inference about the attitudes of such crews in general; however, should the ship commander strongly influence the attitudes of his men, then in all probability such a cluster process would not reflect the over-all set of attitudes. It can be seen that costs and administrative problems would be reduced if cluster sampling can be used of one or several crews rather than a random sample of all such crews in the Navy.

Systematic Sampling

Occasionally the population from which a sample is to be taken is arrayed in a numerical sequence; for example, a series of vouchers or serialized cancelled checks. In this case we can use the technique of systematic sampling rather than simple random. Let us suppose for example that we have a population and that it is desired to select a random sample of 200. A random digit may be selected between 000 and 0049 to indicate the starting voucher number. From this point on, every 50th voucher is selected in a systematic manner. The sample of 200 then constitutes a simple random sample under

these conditions. We must use caution, however, and be certain that the ordering of the population is in no way dependent upon the characteristic examined. For example if every 50th voucher were associated with a particular company, it would be impossible to draw any general inference about all of the vouchers in the population.

Sequential Sampling

Sequential sampling is a technique developed during World War II by Dr. Abraham Wald of the Statistical Research Group at Columbia University. The philosophy of this technique is that often decisions can be made before the entire sample has been examined. The technique involved in this type of sampling is that observations are selected one at a time, with a decision being made at each step, based on the cumulative information after the selection of each observation. Three decisions present themselves (1) that there is not sufficient information to make a decision and an additional observation is necessary; (2) the data are inconsistent with the hypothesis being tested and hence it must be rejected; or (3) the data are consistent with the hypotheses being tested and hence it may be accepted. In other words a decidedly defective population will be apparent after the examination of an extremely small part of a sample, thus it becomes only necessary to sample until our hypotheses is either accepted or rejected. By the use of this technique it is possible to draw a conclusion with a sample size of approximately 50% of that used for simple random sample.

Sampling and Non Sampling Errors

Whenever the technique of sampling is used there is inherent in the process the possibility of encountering sampling errors and sampling biases, however, whenever sampling is performed so that every unit in the statistical universe has a chance of being selected and the probability of selection is known, the errors of sampling can be controlled satisfactorily. This is known as probability sampling.

Some of the important types of sampling errors are encountered when conducting a survey. The average lady over forty reports her age as under forty, a bank teller states his occupation as a banker and significant data may be eliminated from report forms. There are the errors of the non response type specifically a household survey conducted during the daytime would yield a poor estimate of lady truck drivers, an evening follow up of the not at homes would be necessary to overcome this bias. Errors occur in sample selection such as chunk samples conducted for convenience at Joe's bar or the ball park.

The result of a sample survey will hardly ever agree completely with that of a complete enumeration. The difference between the two is known as the sampling error. If a probability sample design is used, it is possible to predetermine the size of sample needed to obtain a specified degree of precision.

Sampling is used in many ways throughout the Department of Defense the results providing important decision making data.

For example sample surveys are conducted of military personnel to learn certain characteristics about them and to find out their opinions and intentions on a wide variety of subjects. This information is used for guidance in formulating personnel policies and actions that have considerable impact on virtually all defense activities.

Recently a sample survey was conducted of Air Force clothing store sales. On the basis of pre-test data it was determined that a sample of 35 out of more than 200 stores world-wide would yield acceptable reliable clothing size distribution. Sales stores were stratified by size of store within climatic regions and samples were selected from each regional listing. The results of the survey yielded accurate size distributions for principal clothing items; the distribution of sales made by climatic region, of sales by item, and by size of store. Such information as this helps to achieve improved purchasing and stockage procedures; smaller inventories, higher inventory turnover; and better service.

By way of summary it can be stated that the appropriate method of statistical sampling is a function of the data desired, the form and availability of the population, the administrative requirements and costs associated with sampling. To determine the method of sampling is the problem of the sampler.

Statistical Inference

In this section we shall discuss a few of the more important techniques of statistical inference and illustrate

some of the various techniques with actual case problems. Before beginning a discussion of statistical inference it is first important that the term be understood. By inference we mean the process of generalizing from a bit or segment of information to the state of affairs in the population from which the segment of information has been obtained. It is a process of inductive reasoning or reasoning from the specific to the general. Stated in another way statistical inference is reasoning from the statistic and applying it to the population--or generalizing from a set of sample results to the population. This is a phase of deductive logic.

There are two basic problems of statistical inference; these are (1) problems of estimation, and (2) testing a hypothesis. For example we may select a sample from a population for the purpose of estimating some parameter or characteristic in the population such as to estimate the proportion of inventory which is valued at less than $20,000. This is known as estimation. Or we may have the problem of observing whether specifications are being met where items are being manufactured and must meet a specific weight or tolerance. This is referred to as <u>testing a hypothesis</u>.

There are several methods for obtaining solutions to the two basic problems of inference and will be discussed in the following paragraphs. The methods to be discussed are as follows: Estimation by (1) <u>the arithmetic mean</u>, (2) <u>proportion</u>, (3) <u>correlation coefficient</u> and (4) regression. The techniques for testing a hypothesis are (1) <u>the arithmetic mean</u>

(2) proportion and (3) correlation coefficient.

The following table indicates the symbols for the population parameters and corresponding sample statistics:

Characteristic	Parameters	Statistics
Arithmetic mean	μ (Mu)	$\bar{X}$
Standard deviation	σ (sigma)	S
Proportion	π (pi)	p
Correlation Coefficient	ρ (rho)	r

Estimation

Estimate the True Proportion

In the following problem it is desired to estimate the true proportion of a population possessing a particular characteristic. Briefly the problem was to conduct an audit and estimate the number of pay records that contain monetary errors.[11] These records are maintained in the officers accounts office of the Department of the Navy, Washington, D. C. For the purpose of this problem a monetary error is defined as the omission or commission of an act which is either definitely prescribed or prohibited by departmental regulations. In this particular problem it was necessary to determine the correct sample size; to determine the percentage of erroneous pay records in the sample; and to estimate the number of erroneous pay records in the universe by statistical inference.

[11] George D. Tracy, Lt. Cdr. USN, "A Pay Record Audit Problem." Unpublished term paper, Department of Statistics, George Washington University, 1958.

Because we are dealing with pay records, and pay record entries govern entitlement to money, we want to insure that the estimate of the error rate in the universe is correct with a 99% degree of confidence.

The following formula is used in the solution of the problem.

$$n = \frac{z^2 \ \pi \ (1- \ \pi \ ) \ N}{(N-1) \ \ E^2 + z^2 \ \pi \ (1- \ \pi \ )}$$

Where:

$z = 2.58$ For 99% confidence limits. Obtained from table I appendix A.
$\pi = .01$ An assumption of the approximate error rate based on a previous NRAO AUDIT.
$E = .01$ (Epsilon) The amount of error that can be tolerated in the statistic "p".
$N = 5{,}920$ The number of pay records in the universe under study.

By substitution of this data in the formula we are able to determine the value for n the sample size.

$$n = \frac{(2.58)^2 \times (.01) \times (1-.01) \times (5{,}920)}{(5{,}920-1) \times (.01)^2 + (2.58)^2 \times (.01) \times (1-.01)}$$

$$n = \frac{(6.66) \times (.01) \times (.99) \times (5{,}920)}{(5{,}919) \times (.0001) + (6.66) \times (.01) \times (.99)}$$

$$n = \frac{(.07) \times (5{,}920)}{(.66)} \qquad = \qquad \frac{414}{.66}$$

$$n = 627$$

Now that the sample size of 627 has been determined the next step is to prepare an alphabetical listing of the names of the officers, for whom pay records are maintained, by running a "money list" using the addressograph plates kept on file for that purpose. Following the preparation of the money

list a starting point should be selected at random and this point checked. Additional names would be checked at an interval of ten names until the 627 names were all checked off. The next step of course is to go to the pay record files and pull the pay records of the 627 names that have been checked. The required sample would be comprised of this group of pay records.

The statistical conclusion that can be made from this sampling process is as follows: Let us assume that the sample contained 12 pay records that had monetary errors. By referring to table 2, appendix A, for 99% confidence limits; where "p" is .02 and the sample size is 627 the table shows that the proportion of the population (π) lies within the range of .01 and .035 which is written $.01 < \pi > .035$. This means that with 99% confidence I can be sure that the percentage of monetary errors in the universe of 5,920 pay records lies between 1% and 3.5%.

There are other approaches to this type of problem. For example if the sample size is large then the following formula yields a fairly accurate set of confidence limits:

$$p - Z\sqrt{p\ (1-p)/n} < \pi < p + 2\sqrt{p\ (1-p)/n}$$

Let us consider the following problem. A random sample of enlisted men indicated that 21% plan to reenlist when their present hitch is up. There were 400 men in the sample. The 99% limits are:

$$.21 - 2.576 \sqrt{.21 \times .79/400} < \pi < .21 = 2.576 \sqrt{.21 \times .79/40}$$

$$.21 -.052 < \pi < .21 + .052$$

$$.158 < \pi < .262$$

From this we may conclude that the 99% confidence limits for the true proportion expecting to reenlist are from .158 to .262,[12] or stated another way from 16% to 26% plan to reenlist

Correlation Coefficient

There are on occasion problems where two variables are analyzed together, and a correlation coefficient is often computed. Because the sample correlation coefficient is subject to some variation it is often of interest to estimate the true population correlation coefficient.

There are precise methods for calculating the desired confidence limits, however, there are tables that can be more easily used to obtain the 95% and 99% confidence limits for the true correlation coefficient. This of course is especiall useful for the non mathematician.

For example a study was run to examine the relationshi between personnel on board, and recurrent maintenance and operation costs for the Marine Corps Supply Depot, Albany, Georgia.[13] A sample of 16 yielded a correlation of .67.

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[12] McCall, op. cit., p. 45.

[13] S. Frank Leis, Major USMC and Jim H. Bolton, Captain USMC. "Correlation of Manpower and Maintenance and Operations Cost of Ten Major Marine Corps Posts and Stations." Unpublished term paper, Dept. of Statistics, George Washington University, 1958.

Consulting table IV appendix A the 95% confidence limits are estimated to be

$$.30 < p < .87$$

The interpretation of these results will be referred to later in our discussion of correlation and regression problems, along with estimation by regression techniques.

<u>Arithmetic Mean</u>

This statistical technique is used when it is desired to estimate the mean of a population, μ , using the mean of our sample $\bar{X}$. If the standard deviation of our population is known the interval estimate for it is determined by the following formula:

$$\bar{X} - Z\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z\frac{\sigma}{\sqrt{n}}$$

The symbol Z is the normal deviate selected from table I to yield the desired degree of confidence. If 95% confidence is desired, Z = 1.96; if 99% confidence is needed Z = 2.576.

For example let us assume a sample of flying hours was obtained yielding the following results:

n (number of observations) = 100

$\bar{X}$ (mean of the sample) = 1.8 hours

the standard deviation of the population is known to be = .6 hours. For a 95% confidence level the interval estimate for (mu) the mean of the population becomes:

$$1.8 - 1.96 \times .6/\sqrt{100} < \mu < 1.8 + 1.96 \times .6/\sqrt{100}$$

$$1.8 - .12 < \mu < 1.8 + .12$$

$$1.68 < \mu < 1.92$$

[illegible] table IV (Appendix A) the 95% confidence limits are estimated to be

$$.70 < \rho < .87$$

The interpretation of these results will be deferred to [illegible] in our discussion of correlation and regression in [illegible] along with [illegible] regression equations.

[illegible]

The statistical technique [illegible] it is desired to estimate [illegible] mean [illegible] using the mean [illegible]. If the standard deviation of our population is [illegible] the interval estimate for [illegible] is determined by the following formula:

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z\frac{\sigma}{\sqrt{n}}$$

Where z is the normal variable obtained from table I [illegible] for the desired degree of confidence. If the confidence is [illegible], z = [illegible]; [illegible] z = [illegible].

For example let us assume a sample of flying hours was obtained yielding the following results:

a. Number of observations = [illegible]

b. Mean of the sample = [illegible] hours.

The standard deviation of the population is known to be [illegible] hours. For a 95% confidence level the interval estimate for [illegible] the mean of the population becomes

[illegible]

[illegible] $< \mu <$ [illegible]

The 95% confidence limits for the true average hours of flying time are 1.68 to 1.92 hours.

Many times, however, μ and σ are not known. In this case the following formula is used $\bar{X}-t\ s/\sqrt{n-1} < \mu < \bar{X}+t\ s/\sqrt{n-1}$. In this formula s = standard deviation of the sample which can easily be computed and "t" is obtained from tables of selected Students "t" values.[14]

Testing a Hypotheses

In practically everything we do in our daily routine of living we are compelled to make decisions. These decisions may be based on experience, impulse and/or scientific evidence. Although there is some diversity of basis for decision making, it is still possible to study and analyze the merits and disadvantages of the various methods of making decisions by evaluating the risks which they force us to assume. A simple hypothesis might be of deciding which road to take, the right or the left, when arriving at a fork in the road or testing a pair of dice to see whether they are biased. The technique of testing a hypothesis is used frequently in the field of quality control. Here certain standards have been set up for production and periodic samples are selected to determine whether or not the items meet specification. A hypothesis is, therefore, a statement which it is desired to reject with a specified degree of confidence, or to accept for lack of sufficient contrary evidence.[15]

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[14] McCall, op. cit., p. 43.

[15] John E. Freund, Modern Elementary Statistics (New York: Prentice-Hall, 1952), p. 189, ff.

There are two types of errors that may be committed in experimental situations of testing a hypotheses. These are called the type one error and the type two error. The type I error occurs when the hypothesis being tested is true and we have rejected it. Type II occurs when the hypothesis being tested is false and we accept it.

Generally in testing a hypothesis the null hypothesis is formed and is represented by the symbol H_o. The null hypothesis is a hypothesis which is formulated for the express purpose of being rejected. It is much easier to prove things wrong than it is to prove things right. If we were asked, for example, to prove that all sheep are white, it would require investigation of every single sheep to determine the color. On the other hand it would take only one black sheep to show that the hypotheses is false. Hence it is much easier to calculate the error which may be committed by rejecting a hypotheses than it is to calculate the error which may be committed by accepting it.

Correlation Coefficient

When a sample correlation coefficient has been computed an interpretation is sometimes desired. In the correlation of manpower and maintenance and operations costs,[16] mentioned earlier, the null hypothesis, that there was no significant correlation between the two variables in the population of personnel on board and maintenance and operations costs was formed.

$$H_o : \rho = 0$$

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[16] Leis and Bolton, op. cit.

To test this hypothesis we refer to table IV appendix A for ρ (rho) = 0 and n = 16. The 95% confidence limits for r are estimated as $-.49 < r < +.49$ hence it is determined that if the calculated correlation coefficient lay outside this range we would have a significant correlation between the variables, with 95% confidence that a true hypothesis was not being rejected. The data for the problem produced an r = .69 for the Marine Corps Supply Depot, Albany; r = .53 for Marine Corps Supply Forwarding Annex San Francisco; and r = .29 for a consolidation of the ten major Marine Corps stations. Since the r = .29 for the ten major stations falls within the range of chance alone $-.49 < r < +.49$ the hypothesis that there was no significant correlation was accepted. For the individual stations, however, the correlation coefficients lie outside the range of chance alone, hence the hypothesis in these cases was rejected. Since there was no correlation for the ten major stations and only a poor correlation for the others there was no need to continue the problem for the purpose of estimation by linear regression techniques. Further study using regression techniques is necessary to determine the relationship of the variables in the case of the other two stations.

To illustrate another case let us use the problem of where a random sample of 200 recruits were given two forms of a test. Form A has been in use for 15 years and took 8 hours to administer. A proposed revision, form B, took only two hours to administer. If the correlation between these two forms was found to be .45, is it safe to conclude at 95% level

that there is better than a chance relationship in the population?[17] Using table IV, appendix A, for = 0 and n = 200, the 95% limits for r are estimated as:

$$-.14 < r < +.14$$

Since the sample correlation of .45 exceeds the sample we can assume with a 95% confidence that the two forms bear some relationship to each other. Further study would be required to determine how much.

Proportion

Certain types of problems are adapted to testing the hypothesis that the true proportion (π) is equal to some specific value. For example an ordnance depot contained a stock pile of 40,000 proximity fuses from World War II. If possible it was desired to use these fuses in a more modern weapons system provided they could produce a specific degree of reliability. If this is possible it could provide a tremendous saving in our defense effort. Since the fuse was only one component out of many in this system a 95% reliability was desired. A sample was taken consisting of 400 fuses, 360 were found acceptable at the 5% level, can the hypothesis that the lot meets requirements be accepted? The mathematics involved in this problem is as follows:

p = 360 the amount of the sample found acceptable

n = 400 the amount in the sample

π = .95 the degree of confidence desired

$$p = 360/400 = .90$$

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[17]McCall, op. cit., p. 52, ff.

consulting table 3, appendix A for n = 400 and $\pi = .95$ the limits for p are:

$$.925 < p < .97$$

The sample proportion is less than .925 hence we reject the hypothesis and conclude with a better than 95% confidence that the lot does not meet the reliability requirements.

<u>Arithmetic Mean</u>

A hypothesis may also be tested by examining the sample mean to see whether it is contained within the limits. If the standard deviation of the population (σ) is known the following formula is used.

$$\mu - z \frac{\sigma}{\sqrt{n}} < \bar{X} < \mu + z \frac{\sigma}{\sqrt{n}}$$

Let's assume a manufacturing firm plans to establish controls, according to government regulations concerning a specified diameter of a production item. The mean diameter should be 21cm with = .5cm. Sample sizes of 16 are chosen periodically and a decision is made regarding the control process at each time. A sample mean of 21.3 has been obtained and the 95% level is considered acceptable. Conclusions:

1. The null Hypothesis that the process is in control is

$$H_o : \mu = 21cm$$

2. Since the amount of error tolerated is =.05 the appropriate value for Z is 1.96, table I.

3. The required limits are:

$$21 - 1.96 \times .5/\sqrt{16} < \bar{X} < 21 + 1.96 \times .5/\sqrt{16}$$

$$21 - .245 < \bar{X} < 21 + .245$$

$$20.755 < \bar{X} < 21.245$$

4. $\overline{X} = 21.3$ cm. falls outside of the limits, hence we reject our hypothesis with better than 95% confidence and examine the process.

When the σ is not known the following formula is used

$$\mu - t\ S/\sqrt{n-1} < \overline{X} < \mu + t\ S/\sqrt{n-1}$$

the value for "t" is obtained from Students table for values of "t".

Correlation and Regression

Correlation analysis is a statistical process of discovering and measuring functional relations of two or more series of data in order to ascertain the extent to which the variables are correlated. In other words to determine to what extent they move together or oppositely. Stated another way correlation involves a study of the relationships existing between two or more variables, while regression may be defined as the prediction of one variable by means of one or more addition variables.

To illustrate an approach to this technique when only two variables are involved would be to plot our observed data on graph paper using an X and Y axis for each of the variables, for example smoking versus lung cancer. If the observations were scattered as seen below in figure 2 we would assume that no correlation exists,

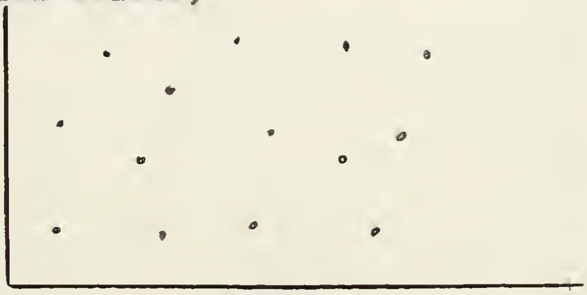

fig. 2

however, in figure 3 lung cancer increase with smoking, not a significant correlation however.

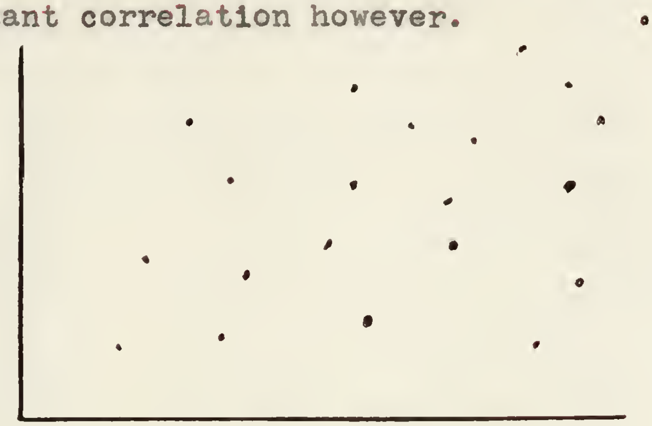

fig. 3

The observations in figure 4 show a strong positive correlation,

fig. 4

while figure 5 describes a strong negative correlation.

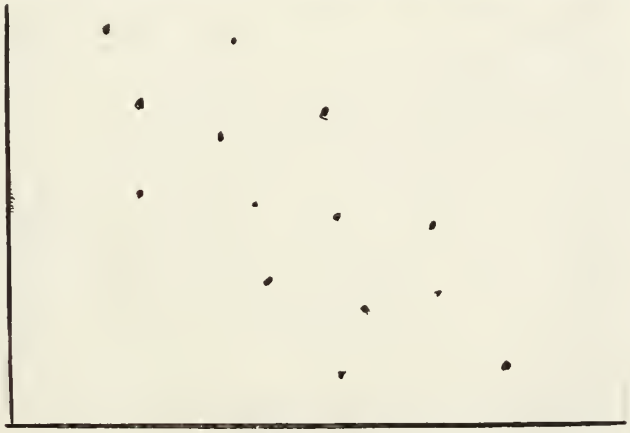

fig. 5

For linear relationships, correlations which describe a straight line relationship between the two variables, a measure of the degree of this correlation has been developed by Karl Pearson. This measure is known as the Pearson product-moment correlation coefficient. This measures only the linear relationship between the variables and is written:

$$r^2 = \frac{\left[n \quad XY - \quad X \quad Y\right]^2}{\left[n \sum X^2 - (\sum X)^2\right]\left[n \sum Y^2 - (\sum Y)^2\right]}$$

To illustrate the concept of correlation let us consider a typical problem and one that is of continuing interest to the comptroller from the standpoint of budget prediction and cost control. The particular problem under consideration was formulated for the expressed purpose of attempting to find a method for predicting the recurrent station operations and maintenance costs for Marine Corps posts and stations. In this particular case the data did not produce significant results when all of the stations were consolidated indicating this approach was not a useful predictive device under this particular set of circumstances, however, correlations were evident when the data for each individual post or station was computed and analyzed individually. For the purpose of illustrating this technique we shall use the data for the Marine Corps Supply Depot, Albany, Georgia.[18] The original data are omitted but the following sums were obtained from 16 observations. "X" represents personnel on board and

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[18]Leis and Bolton, op. cit.

"Y" represents recurrent operations and maintenance costs in dollars.

$$\Sigma X = 54,900$$

$$\Sigma X^2 = 190,508,002$$

$$\Sigma Y = 61,959^{x10^2}$$

$$\Sigma Y^2 = 241,323,927^{x10^4}$$

$$\Sigma XY = 213,754,156^{x10^2}$$

$$(\Sigma X)^2 = 3,114,010,000$$

$$(\Sigma Y)^2 = 3,838,917,681^{x10^4}$$

$$\Sigma X \, \Sigma Y = 3,401,594,100^{x10^2}$$

$$n = 16$$

$$\bar{X} = 3,431.25$$

$$\bar{Y} = 387,243.75$$

Substitution of these data in the formula page 53 provides the following:

$$r^2 = \frac{\left[16\ (213,754,156^{x10^2}) - 3,401,549,100^{x10^2}\right]^2}{\left[16\ (190,508,002) - 3,114,010,000\right]\left[16\ (241,323,927^{x10^4}) - 3,838,917,681^{x10^4}\right]}$$

$r^2 = .45138877$

$r = .671854$ (this is the square root of r^2)

The r^2 is called the coefficient of determination and is sometimes used as a measure of agreement. It gives the percentage of the variation in one variable which may be accounted for by the variation in the other variable. In this case r^2 is .45139.

This indicates that the variability in recurrent operations and maintenance costs, 45.12% may be accounted for by the variability in personnel on board. The remaining 54.88% is due to other factors.

Since our purpose in this problem is to utilize one variable to predict the other it now becomes necessary to extend the correlation concept into regression. We shall consider only the simplest of relationships the straight line or linear relationship at this time. In the population, the true linear relationship is represented by the equation

$$Y = \alpha + \beta X$$

where α and β are constants, α being the Y intercept and β being the slope of the regression line. The regression line obtained from the sample, is $Y = a + bX$. a and b are the sample estimates for the parameters α and β. To obtain a and b the following formula is used:

$$b = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2}$$

$$a = \bar{Y} - b\bar{X}$$

Substituting the data in the formula we have:

$$b = \frac{16\ (213{,}754{,}156^{x10^2}) - 3{,}401{,}549{,}100^{x10^2}}{16\ (190{,}508{,}002) - 3{,}114{,}010{,}000}$$

$$b = \frac{1{,}157{,}338^{x10^2}}{2{,}132{,}377}$$

$$b = 54.27454$$

$$a = 387{,}243.75 - 54.27454\ (3431.25)$$

$$a = 387{,}243.75 - 186{,}229.52$$

$$a = 201{,}014.23$$

$$Y = a + bX$$

$$Y = 201{,}014.23 + 54.2745X$$

On the basis of these answers we can now predict the value for Y, the recurrent operations and maintenance cost, for a given value of X, the personnel on board. If we plot these data on a graph we can see that there is a fair positive correlation, however, it is felt this correlation is not quite strong enough to be used with reliability in estimating budget requirements.

(Figure 6 is on following page)

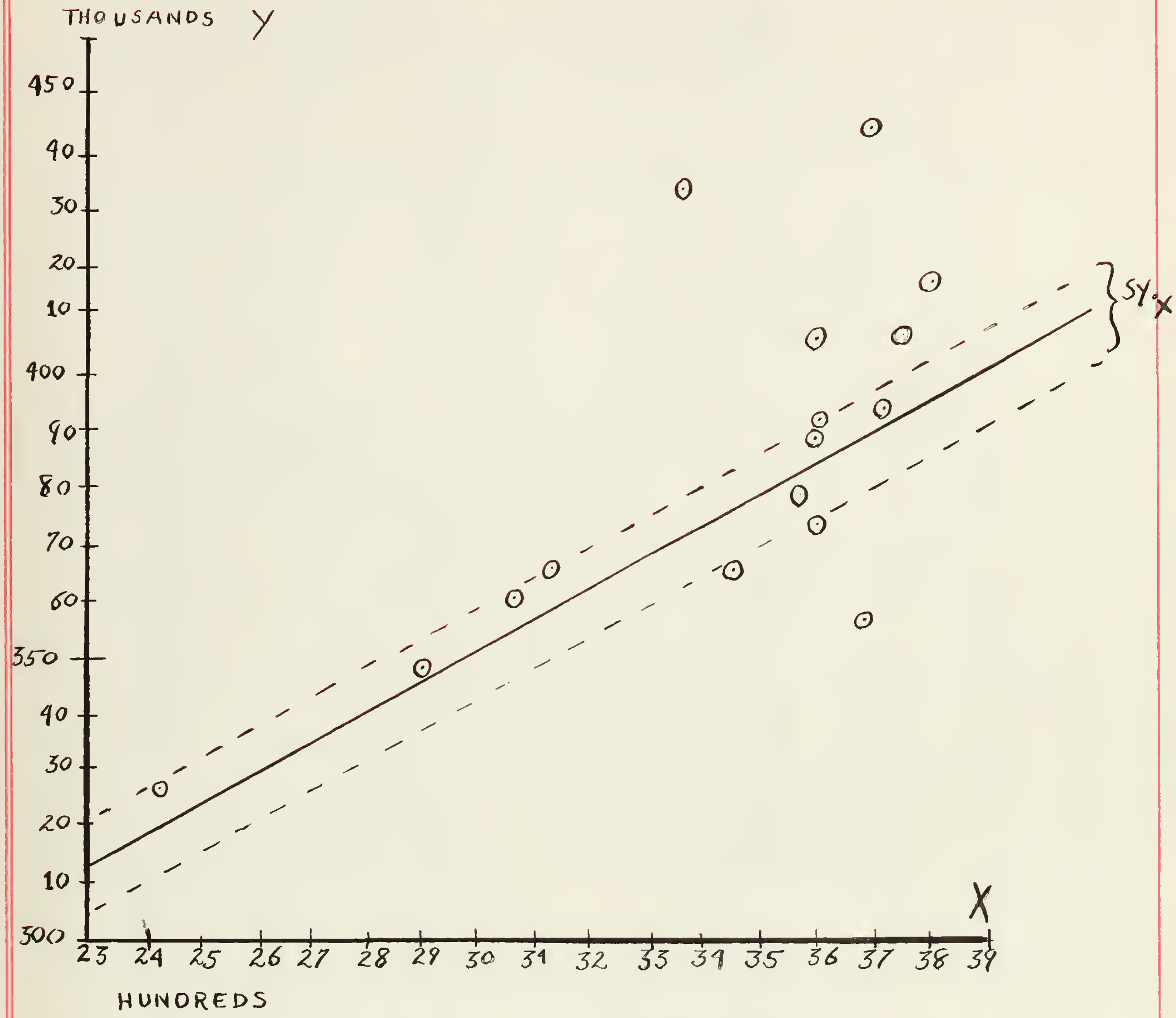

fig. 6

Our next concern is to determine the standard error of the estimate. This is a measure or value of the regression line. Of course the better the correlation the closer the dots will be clustered about the regression line. This measure is actually an estimate of the standard deviation of the observed points about the regression line. The sample formula

for this measure is as follows:

$$Sy.x = \sqrt{\frac{(1 - r^2)\, y^2}{n - 2}} \qquad \text{Where } \Sigma y^2 = \Sigma y^2 - \frac{(\Sigma Y)^2}{n}$$

$$Sy.x = \sqrt{\frac{(1 - .45)\, 86,973.25^{x10^4}}{14}} \qquad \Sigma y^2 = 241,323,927^{x10^4} - \frac{3,838,417,681^{x10^4}}{16}$$

$$Sy.x = \sqrt{34,168,062.50} \qquad \Sigma y^2 = 86,973,.25^{x10^4}$$

$$Sy.x = 5845.34$$

This standard error is relatively small, being roughly 2%. This result cannot be attributed to a strong correlation, but mainly to the relatively small range of values of Y. The range of values is approximately 33%.

Conclusion

In conclusion of this section it is well to point out that the statistical technique were only a few of the many different applications of mathematical techniques that can be applied to meet the particular needs of the situation. The point that this author would like for the reader to carry away is that the mathematics involved is merely the substitution of numbers for letters and simple arithmetic. It is hoped that the examples presented to illustrate the statistical methods illustrate the variety of control type problems that can be solved by the techniques. In addition to the problems already presented, correlation and regression has been used to compare direct labor hours to product overhead in dollars for naval ship yards. Maintenance and operations costs have been studied with aircraft on board for air stations producing some

interesting values for r. The sampling techniques discussed in the first part of this section have been used to develop a work sampling method for naval shipyards. Correlations have been determined for such additional problems as a comparison of sick leave taken by various categories of civil service personnel, and a comparison of M and O costs to plant property. In the field of work measurement these statistical techniques are the major tools.

CHAPTER IV

LINEAR PROGRAMING

Within the everyday activities of executives and the constant requirement for their decisions there exists mathematical tools, other than those just discussed in chapter III, which he may use to assist him. In recent years mathematicians have worked out a number of new procedures which make it possible for management to solve a wide variety of important problems much faster, more easily, and more accurately than ever before. These procedures have been called "linear programing" and sometimes "mathematical programing."

> Mathematical programing is not just an improved way of getting certain jobs done. It is in every sense a new way. It is new in the sense that double entry booking was new in the Middle Ages, or that mechanization in the office was new earlier in this century, or that automation in the plant is new today. Because mathematical programing is so new, the gap between the scientist and the businessman--between the researcher and the user--has not yet been bridged. Mathematical programing has made the news, but few businessmen really understand how it can be of use in their own companies.[1]

Within the planning and programing activities of the Armed Services there are many problems which are concerned with the most efficient use of men and materiel. The solutions

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[1] Alexander Henderson and Robert Schlaifer, "Mathematical Programing," Harvard Business Review (May June, 1954), p.73.

to these problems contribute much to successful operation and decision making within the various branches of the Armed Services.

The development and application of methods for determining solutions to such programming problems is a matter of continuing research. With the cooperation of mathematicians, economists and other scientists, both in and out of the Armed Services, mathematical procedures have been devised that solve an important class of programing problems.

Basic Principles

Problems in the planning and programing area are characterized by limitations and constraints imposed upon the availabilities of the items under consideration. For example, the number of men and dollars available to operate a program may be limited.

These problems are capable of producing many possible solutions. That is the constraints of such problems can be satisfied in many different ways. The selection of the most acceptable or desirable solution ordinarily calls for judgment and experience.

What the mathematical methods of linear programing do is to reduce the whole procedure to a simple, definite routine. There is a rule for finding a program to start with; there is a rule for finding the successive changes that will increase the profits or lower the costs, and there is a rule for following through all the repercussions of each change. What is more, it is absolutely certain that if these rules are

followed, they will lead to the best possible program; and it will be perfectly clear when the best possible program has been found. It is because the procedure follows definite rules that it can be taught to clerical personnel or handed over to automatic computers.[2] In other words, by a systematic procedure, a single efficient solution is chosen that meets the conditions of the problem. A solution is efficient if it accomplishes a specified objective of the problem. The efficient solution may, according to the problem, minimize dollar cost or expenditure of a critical item, or maximize some measure of combat effectiveness. Such a solution that can solve both the constraints of the problem and the given objective is an optimum solution.

The computational methods of linear programing are very general and are applicable to a large variety of problems such as the following:

1. The most efficient routing (in terms of transportation cost, or time, or requirements for use of ships or aircraft) of material from a group of supply or production points to a group of storage or user points.

2. The efficient and economical use of scarce raw materials, or machines, to produce the maximum output of critical goods.

3. The best assignment of output production of military items, such as planes, between competing demands of combat and training missions.

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[2] _Ibid._, p. 75.

4. The minimum cost of contract awards to meet a given supply requirement from among a group of bidders quoting various prices, maxima and minima production for various items, and other quantity or price restrictions as to bids.

Basic to all of the problems is the matter of limited recourses which must be shared among a number of competing demands, and as can be seen from the above problems, these limited recourses may be manpower, or capital, or material, or equipment, or possibly a combination of all of these.

Mathematical programing, of which linear programing is but one phase, has been termed a restatement of the central formal problem of economics--that of allocating scarce resources so as to maximize the attainment of some predetermined objective--in a form which is designed to be useful in making practical decisions in business and economic affairs. The accent here is on the word "practical." The theory and principles that have led to the development of the mathematical programing techniques coming increasingly into business and military use may well be far beyond the comprehension of the non-mathematical layman, but there is nothing theoretical about their applications; a glance at the four typical problems listed above should dispel any doubt on that score. It is no longer premature to say that mathematical or linear programing has proved its worth . . . for finding optimum economic programs. . . . It is a practical tool for the comptroller and all phases of business planning.[3]

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[3]R. Dorfman, "Mathematical, or 'Linear' Programming," <u>American Economic Review</u>, XLIII (December, 1953), p. 797.

Problems

A simple example of an application of linear programing is probably the best manner in which to explain and demonstrate just what it can do. Of the many types of problems that lend themselves to the methods of linear programing for solution are a group termed transportation problems. These problems call for the shipping of a supply of an item from a group of depots to a group of receiving stations. Each depot has a limited amount of the item, while each station has a specific requirement for the item. Usually there are many routings which will supply each station with exactly the required amount of the item. The problem is to determine the routing which not only fulfills the requirements but also minimizes some measure of the program cost. For example, the objective might be to minimize one of the following: the total dollar cost, the total number of miles of the shipping schedule, or the total time the items are in transit.[4]

A fictitious example of a routing problem follows. The objective in this case is to minimize the total ton-miles. Let us assume that Lockbourne AFB at Columbus, Ohio has been testing a large item of equipment, weighing a ton, for the B-47. It is now desired that this equipment be tried at other bases. Five each are required by March AFB at Riverside, California; Davis-Monthan AFB at Tucson, Arizona; and McConnell

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[4] The Application of Linear Programming Techniques to Air Force Problems: A Non-technical Discussion (Washington, D. C.: Directorate of Management Analysis, DCS/Comptroller, Hdqts., USAF, 17 December 1954), p. 10.

AFB at Wichita, Kansas. Pinecastle AFB at Orlando, Florida, and MacDill AFB at Tampa, Florida, each need three. To supply these 21 items Lockbourne AFB at Columbus, Ohio can ship 8; Oklahoma City Depot has 8, and Warner-Robins AFB at Macon, Georgia has 5. The items of equipment are to be air lifted to their destinations. The problem is to determine the routing which fulfills the requirements and minimizes the total ton miles. All the foregoing information together with the approximate air distances is summarized in the following table:[5]

	Items Available	Required Items	MacDill	March	Davis Monthan	McConnell	Pinecastle
			3	5	5	5	3
Oklahoma City	8		938	1030	824	136	995
Macon	5		346	1818	1416	806	296
Columbus	8		905	1795	1590	716	854

Distance in Miles

A first attempt at a solution might go like this: Try to fill each requirement from the nearest source. For example, let McConnell get its 5 items and Davis-Monthan 3 of its 5 items from Oklahoma City. The requirements for the Florida bases should be met, as far as possible, from Macon. Let Pinecastle get its 3 and MacDill 2 of its 3 items from Macon. The rest of the requirements must be met from Columbus. This shipping schedule has a total of 17,792 ton-miles and is

[5]*Ibid.*, p. 11, ff.

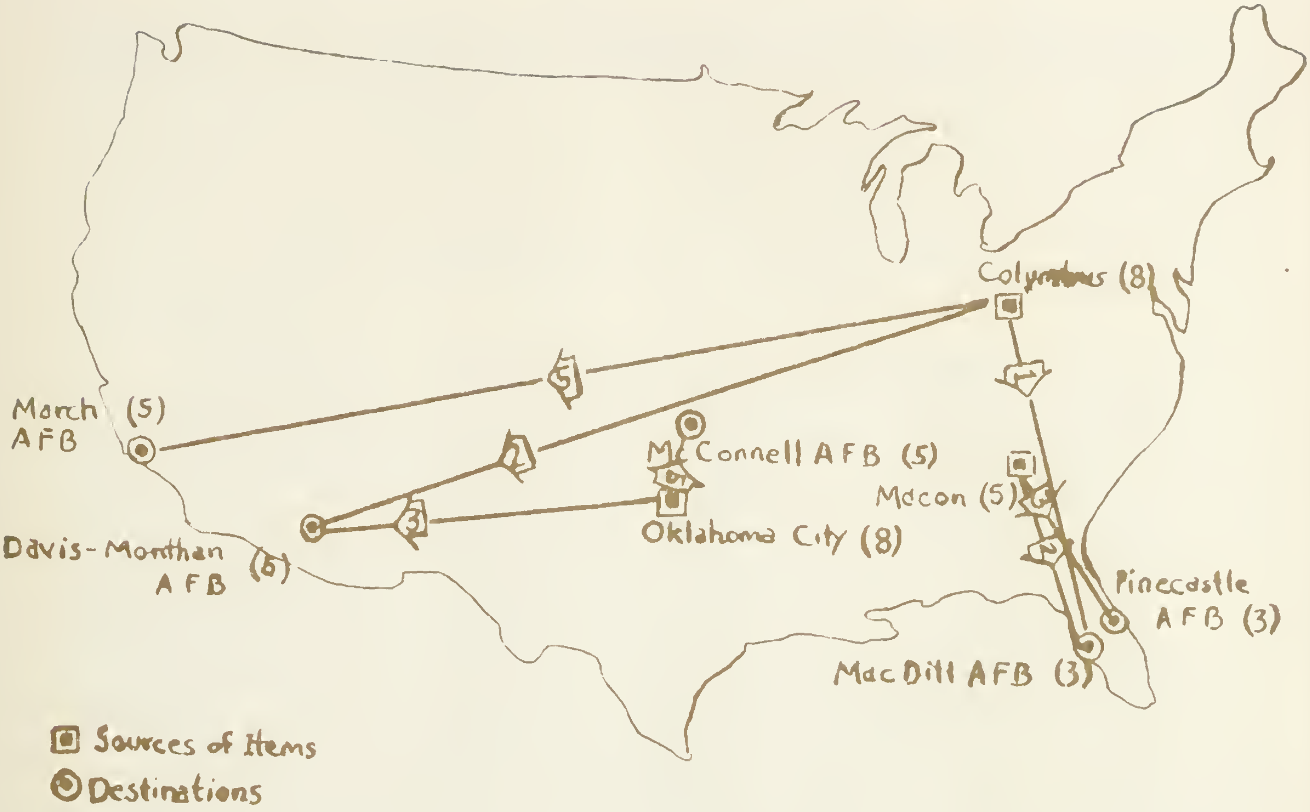
illustrated in figure 7, page 67.

There are many other possible routings. The minimum solution, obtained by the methods of Linear Programing, has a value of 16,864 ton-miles. The computational procedure starts with any solution to the problem--such as the one desired above--and in a systematic manner obtains better solutions until the optimum one has been determined. This Linear Programing procedure guarantees that the optimum will be found. For this simple example, it is not difficult, given the number of ton-miles in the optimum solution, to discover the corresponding routing. Without this information, one would not, in general, be able to determine just which solution is optimum. This is especially true for larger problems where solutions by hand becomes impractical.

If in attempting to take advantage of the shortest distance appearing in the problem by trying to fill all of McConnell's needs from Oklahoma City--the resulting solution would be less efficient. This fact serves to emphasize that what at first appears to be a logical approach to problems of this nature is not necessarily the best one.

Another simple example of a transportation type problem is as follows: Suppose that a regional freight car distributor must send empties from three divisions on which car surpluses are predicted, to five other divisions exactly equal to the total surpluses, though this is not a limitation for other than simplifying the example. Conditions can best be expressed and summarized in the following table:

SAMPLE SOLUTION



MINIMUM SOLUTION

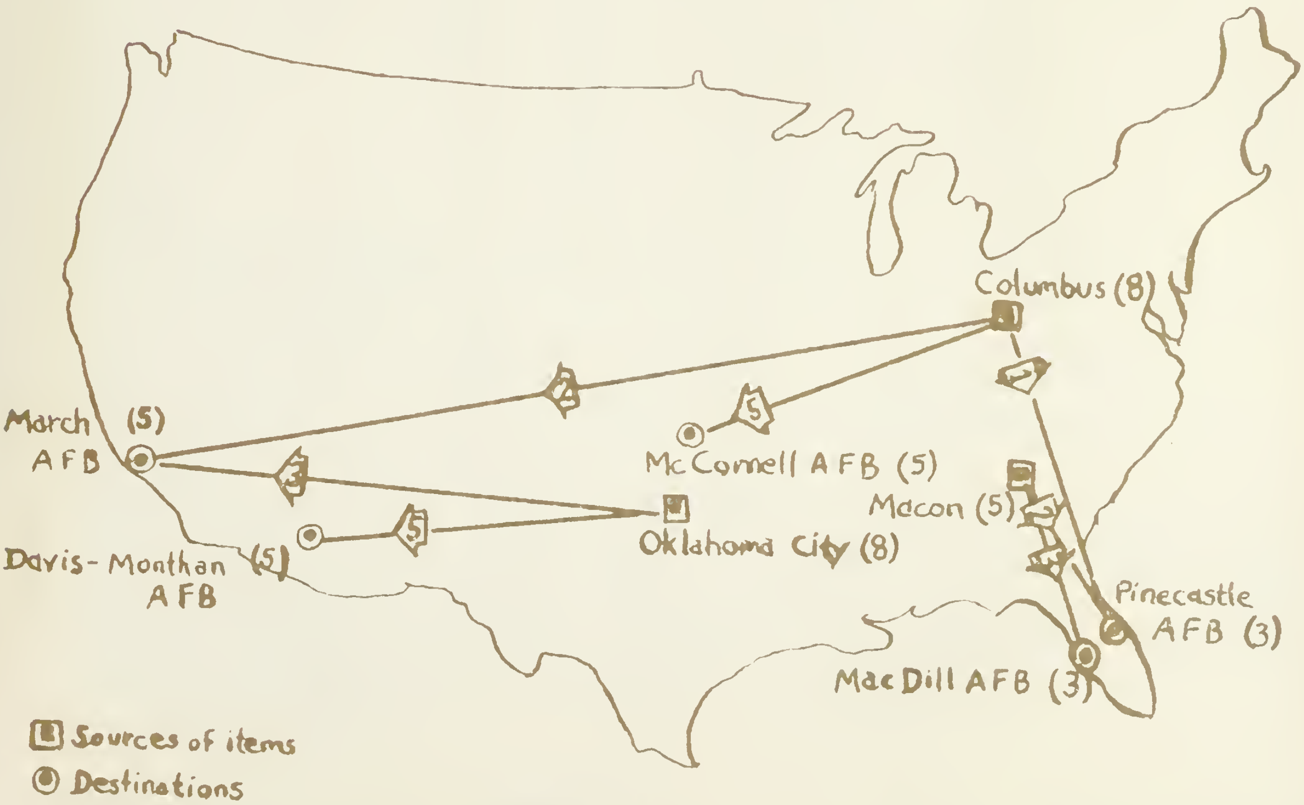


Figure 7

Figure 7

Origin	Destination					Surpluses
	D1	D2	D3	D4	D5	
S1						9
S2						4
S3						8
Shortages	3	5	4	6	3	21

It should be noted at this point that the problems given have necessarily been extremely simple; so simple, in fact, as to be susceptible of solution by trial and error if not by inspection. But if the examples were too extensive, they would fail as instructive and readily comprehendable examples. The table merely states that Division S1 has a surplus of 9 cars; D2 a shortage of 5 cars, etc. The cost of shipping an empty car from a shipping point to a destination is tabled as follows:

Origin	Destination				
	D1	D2	D3	D4	D5
S1	10	20	5	9	10
S2	2	10	8	30	6
S3	1	20	7	10	4

The table indicates that it costs $10 to send an empty car from Division S1 to D1; $30 to send a car from S2 to D4, etc. Now, while it is possible to obtain a feasible solution by inspection, it is not possible to know whether the solution selected is the least cost solution, or what greater cost, if any, the solution has than the least cost. Linear Programing offers a method not only of obtaining the least cost solution,

but also of ensuring that it is in fact the least cost. Appendix "C" gives the solution to this problem. The minimum cost is $150; one might find it interesting to make up a feasible solution and cost to see how it compares with the $150 figure.[6]

When one realizes that the same technique is capable of solving problems involving a table 300 x 500 as easily as the 3 x 5 table shown in the example, the advantages are obvious. The time required is of course greater, but the technique is identical, and with the aid of electronic computers the time required is a matter of hours or minutes.

Military Use

Within the Armed Services it is often desired to make an efficient deployment of combat aircraft to competing activities or missions given a production schedule. Those high priority missions of course have first claim on the available aircraft. It is necessary that we provide the maximum amount of aircraft for combat, however, we want to simultaneously allocate enough aircraft to advanced flying schools to train crews to fly them. Another objective is to minimize the surplus of crews. Results to be computed on a world wide basis and then by area. These examples are typical of the type of problem that faces the military executive that can be solved by linear programing procedures.

These problems of course are mainly of an operational

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[6]"Operational Research in Distributing Empty Cars," <u>Railway Age</u>, CXXXIV (April 20, 1953), pp. 73-4.

nature, however, in the end analysis cost is involved and in this the comptroller becomes vitally interested. More specifically the comptroller would be interested in problems concerning contract awards. For example: Whenever it is desired to procure items from civilian sources, producers of the items must be invited to participate in the bidding for contracts. The individual manufacturer submits bids in which he states:

1. The price per unit of article or articles.

2. The maximum (and minimum, if any) quantity of each item that can be produced at the stated price, and

3. Any other conditions he wishes to impose. The bid reflects the manufacturer's desire for profit, his guess about the other fellow's bid, and his own peculiar limitations.

The comptroller ultimately having to pay for the procurement of specific quantities of related articles, must see that contracts are awarded in such a way that the total dollar cost to the government is at a minimum.

In evaluating bids the comptroller's shop must add shipping and other related costs to each of the bidders' quoted prices. Similarly, any savings that could be effected by agreeing to certain conditions are subtracted, e.g. a discount may be allowed for payments made within a certain time. Once the contracts have been awarded, the procurement office must be ready to demonstrate--even to the satisfaction of the losing bidders--that the total cost to the activity of all contracts was the least possible.

The Bureau of Ships, Department of the Navy, has applied Linear Programing to yet another problem, most closely related to the economics problem of the demand too great and the resources too few. Electronics equipments must be allocated to fill all authorized allowance requirements aboard the ships and on shore stations of the Naval Establishment. Equipments are seldom available in sufficient quantities to meet all requirements. In such cases, priority of ship types and projects is one of the controlling factors. The use of new programing techniques will give an optimum use factor to the equipments available, and hence to the ships they go on, than has formerly been possible.

Limitations and Other Methods

Limitations of the technique of applying Linear Programing are two-fold in nature: mathematical and practical. The practical difficulties are those commonly associated with applying any pure science to business and financial problems. One finds it difficult to express in suitable measurable terms the objectives and controlling factors, or constraints; mathematical programing has no monopoly on this difficulty, and it is of course a difficulty associated with the particular operation under investigation and study, not with the technique being employed to investigate it. Secondly, even when one is able to find suitable and measurable terms for the operation, it may well be exceedingly difficult to determine proper numerical values for the coefficients of the terms. Like any other problem, if a more precise answer is needed, more precise

data must be used, and that in turn entails a thorough and detailed study of the problem. Finally, there is the computational labor required to solve the problem, once it has been set up in measurable terms with proper coefficients. The very fact that mathematical programing techniques are being used to investigate a given operation is indication in itself that the problem is quite complex, with many variables and constraints, such that it does not lend itself to quick solution by other means, or by inspection. We must necessarily expect then in cases of application of the technique that computational labor may be immense, even for electronic clauclators; must accept that fact as the nature of the beast; and be thankful that there is a technique to help us where until recently there was nothing but educated guessing. "Thus the practical difficulties encountered in applying linear programing are in a sense an indication of the level of difficulty of the problems which linear programing can treat."[7]

There are many methods used in solving problems by linear or mathematical programing methods. Some types of problems lend themselves to specific forms of solution better than they do to others. At the present time theoretical work in methods of linear programing is ahead of the application of these techniques, in other words, more is presently known about the subject than is put into actual practice.

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[7]Joseph O. Harrison, Jr., "Linear Programming and Operations Research," Operations Research for Management, ed. Joseph F. McCloskey and Florence N. Trefethan (Baltimore: The Johns Hopkins Press, 1954), p. 237.

It is impossible within the scope of this paper to discuss in detail all of the presently existing methods. The following are a few of the procedures used and a selected bibliography for reference:

1. Transportation Procedure[8]
2. The Simplex Method[9]
3. Profit-Preference Procedure[10]
4. The Solution by a System of Ordinary Differential Equations[11]
5. The Dual Method[12]

Perhaps the best known method is the Simplex Method. This method requires that a mathematical solution is feasible at the start, however, it need not necessarily be a practical solution from a business point of view. A sequence of feasible solutions are computed; each one is computed from its predecessor in a manner such that the best answer is gradually

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[8]Henderson and Schlaifer, p. 95.

[9]*Ibid.*, p. 100.

[10]A. Charnes, W. W. Cooper, D. Farr, "Linear Programming and Profit-Preference Scheduling for a Manufacturing Firm," *Journal of the Operations Research Society of America*, I (May, 1953), pp. 114-29.

[11]G. W. Brown and J. Von Neuman, "Solutions of Games by Differential Equations," *Contributions to the Theory of Games* (Princeton, N. J.: Princeton University Press, 1950) ed. by Harry W. Kuhn and A. W. Tucker, pp. 73-9.

[12]C. E. Lemke, "The Dual Method of Solving the Linear Programming Problem," *Naval Research Logistics Quarterly*, I (March, 1954), pp. 36-47.

approached and that a previous solution is never repeated. There are a finite number of solutions between the range of the original one and the optimum. These operations are carried out by simple repetitive and tedious arithmetic steps. It is possible in the case of simple problems to use the hand method or desk calculators, however, for large problems electronic computers would be required.

The easiest procedure and the one I prefer is the Transportation Problem Procedure. The name of this procedure is somewhat misleading and has the connotation that this procedure can only be used for transportation problems, however, there are a multitude of problems that can be solved by the procedure. It acquired this misleading term by being first applied to determine a lowest cost shipping program. Appendix "C" contains instructions for its use and provides step by step instructions for solving a problem concerning empty freight cars.

The other methods listed are usually applied to specific types of problems for which they are better suited than the simplex method. By and large, however, experience has shown that the Simplex method is the most probable method to use.

Summary

In summary we can state briefly that Linear Programing is a mathematical method which can be applied to a variety of problems both civilian and military--and of particular use to the comptroller in those areas of cost; and the techniques of Linear Programing can be used to find efficient solutions to these problems.

CHAPTER V

OPERATIONS RESEARCH

There is a new concept in management that recently has become of vital interest to the comptroller. It is called operations research. The basic characteristic of operations research is the fact that the majority of the practitioners were trained in the basic sciences rather than in engineering or administration. Some other important characteristics are: that it is concerned with research on the operations of the whole organization; the optimization of operations in a manner that brings about greater assurance of both short and long range health for the organization; the newest scientific methods and techniques are applied to the proble; synthesis and extension of the methods and techniques of the older management sciences; analytical models are developed and used in a manner common to the basic sciences; experimental operations are designed and used in a manner that give an insight into the behavior of actual operations; and the use of integrated and creative multi-disciplinary team research to solve complex operational problems.[1]

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[1] Joseph F. McCloskey and Florence N. Trefethen (ed.), <u>Operations Research for Management</u> (Baltimore: Johns Hopkins Press, 1954), p. xiv.

Operations Research Defined

By definition Operations Research is:

> The prediction and comparison of the values, effectivenesses, and costs of a set of proposed alternative courses of action involving man-machine systems. To do this, it uses a model of the action that has been developed analytically by a logical and when feasible, a mathematical methodology. The values of the basic action parameters are derived from historical analysis of past actions or from designed operational experiments. Most importantly, because all human and machine factors are meant to be included, an estimate of the uncertainty in the predicted outcome and in the values, effectivenesses, and costs of the proposed action is provided.[2]

In defining and attempting to understand operations research it is often confused with statistics, especially as applied to the body of specific techniques based upon probability theory. The operations research analyst does, however, use statistical methods when applicable to finding the solution of the particular problem at hand. Because of the wide spread use of statistical technique and method used in the solution of many problems by the operational research analyst, I felt that a discussion of this technique was appropriate in this paper dealing primarily with statistical method. Although the operational research analyst does use statistical techniques often he is not restricted to them. Statistics is concerned primarily with the relations between numbers, while operations research is concerned with reaching an understanding of the operation--of the underlying physical system which the numbers represent. This could cause a significant difference in the results as well as the approach. For example in a

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[2] *Ibid*., p. xxiii.

recent advertising study, the operations research team found the key to characterizing the way in which advertising affected consumers in the results of a series of "split-run" tests; earlier results had been presumed useless after statistical methods, such as analysis of variance and multiple regression, had failed to show meaningful conclusions.

Operations research is also sometimes confused with accounting. This is particularly true when the control aspects of accounting are being considered, however, it should be noted that accounting is one of the principal sources of data to support an operations research study. Within the analysis and construction of measures of control and the functions of operations research there is some overlap as well as a mutual interest on the part of the accountant and the operations research analysts. It has been said in comparing the two fields that the accountant has served a useful purpose in bringing the importance of control measures to the attention of business management, while operations research has shown ability in building new methods for developing and implementing these concepts of control. Other services such as marketing research, engineering, and the industrial engineering field are sometimes confused with operations research. Perhaps the most significant difference between operations research and these other services lies in the type of people employed. Operations research people are scientists, not experts. Their value is not in their knowledge or business experience but rather in their attitude and methodology. It should be noted also at this point

that operations research is not in competition with the other services, but rather compliments the other services and frequently serves to integrate other information, and uses the expert opinion and factual data provided by other services in an organized and systematic analysis. A well organized operations research group should have available the services and counsel of experts in the other fields for the most effective joint attack on management problems.[3]

Types of Problems

In chapter II of this paper it was noted that operations research had its beginning primarily in solving military problems of a tactical and strategic nature. It is well at this point to list some areas of application other than those of a strictly military nature. Today operations research is used to solve such diverse business problems as directing salesmen to the right accounts at the right time, dividing the advertising budget in the most effective way, establishing equitable bonus systems, improving inventory and reordering policies, planning minimum-cost production schedules, and estimating the amount of clerical help needed for a new operation in order to accomplish these tasks. Operations research analysis uses the mental processes and the methodologies which we have come to associate with the research work of the physicist, the chemist, and the biologist--what has come to be called "the scientific method."

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[3]Cyril C. Herrmann and John F. Magee, "Operations Research for Management," Harvard Business Review (July-August, 1953), p. 108.

The Scientific Method

The basic premise underlying the scientific method is the belief that phenomena have a cause, and it is the scientist's contention that by hard work the mechanism or system underlying the observed facts can be discovered and used to the investigator's own best advantage.

The scientific method calls for a special mental attitude, that is a reverence for facts, facts that not only explain the result but those that explain the causes also. When the executive looks at sales figures, he sees them in terms of the success of the sales campaign and how it will affect profits, while on the other hand the scientist looks at these same figures and seeks a clue to the fundamental behavior pattern of the customers. By the process of induction he tentatively formulates a theoretical system or mechanism; then by an inverse process of induction he can determine what phenomena should take place. He then checks these against the observed facts. His test is simply a matter of determining if quantitative data is produced such as can be used for predicting how the customers will behave. For example:

> In a company manufacturing specialty products, examination of account records showed that customer behavior could be accurately described as a time-dependent Poisson process--a type of phenomenon found widely in nature, from problems in biology to nuclear physics. This concept yielded the key to establishing measures of the efficiency of the salesmen's work and of the effect of the promotion in building sales. On this basis a new method of directing promotional salesmen to appropriate accounts was constructed--and then tested by careful experiments, to see if sales increases resulted at less than proportionate

increases in cost (the results in this case were spectacular: an overall sales rise in six figures, and a corresponding gain in net profits).[4]

Basic Concepts

There are four basic concepts important to the technique of operations research. These are (1) the model, (2) the measure of effectiveness, (3) the necessity for decision, and (4) the role of experimentation.

The first concept the model is a simplified representation of an operation, containing only those aspects which are of primary importance to the problem under study. The model greatly facilitates the investigations of operations. The operations research analysts uses a model in much the same manner as the aeronautical engineer uses a model airplane to investigate the aerodynamic properties in a wind tunnel; or the accountant who uses the accounting model, a simplified representation on paper, in the form of accounts and ledgers, of the flow of goods and services through a business enterprise, and thus measures the rate of flow, the values produced, and the performance achieved. Many models are also used in physics. These are sets of mathematical equations.

Operations research models are either exact or probabilistic: (1) The exact model is used in cases where chance plays a small role and the effect of a given action is easily and reasonably determined; such as in long range production scheduling in the face of known demand. This type

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[4] *Ibid.*, p. 101, ff.

of model is accurate enough so that except for some very unusual circumstance, over the long run planned and actual production will be reasonably close. (2) The probabilistic model explicitly recognizes uncertainty such as would be the case in advertising where unpredicability of the consumer plays a great role. The methods developed for physical problems involving mass behavior under random conditions can be applied easily and with great value to operations.

The second basic concept, measure of effectiveness, is a measure of the extent that the operation is attaining its goal. One measure of effectiveness if the return on investment or the net dollar profit.

The third concept is that of decision making, and the existence of alternative courses of action. The objective of operations research is to clarify the relation between the several courses of action, and to determine their outcomes, and to decide which course of action measures up best in terms of the company goal.

The fourth concept concerns the role of experimentation. Operations research is essentially the application of science to the study of operations. The theory, or model, is built up from observed data or from experience. There are two kinds of experiments: the first is designed simply to get information, the other type is designed to test the validity of conclusions.

The application of the basic concepts to management problems present some areas of difficulty. The first problem

is the choosing of the initial area of investigation, selecting the personnel to conduct the work and developing organizational plans for future growth. In commencing an operations research study or investigation there should be an opportunity for decision between alternative courses of action. The problem must be capable of quantitative measurement and study. The possibility for data collection must be present, and the possibility of evaluating results must be present. Management and this applies to comptrollers in the military have two alternatives in obtaining operations research people. The use of outside consultants may be resorted to or train members within the organization. Both alternatives have their advantages and disadvantages. Sometimes a compromise is effective; that is, having an outside group assist in getting an internal organization started.

Operations research should only be undertaken with the intention of continuing and expanding its use. The investment in knowledge and methodology on the staff is too valuable a commodity to throw away.

Evaluation

Case histories have shown that the techniques of operations research provides a basis for objective analysis of operating problems. These studies are quantitative and provide the basis for sound estimates of requirements, objectives and goals, and a basis for planning and decision making. Through operations research there has been a technique for the application of organized thinking to data already existing

within the organization, and for the introduction of new concepts and new methods of analysis.

However, we must make this observation. Operations research is not a cure-all for every management ill, nor is it a source of automatic decisions. It is limited to the study of tangible, measurable factors. The many factors affecting management decisions that are intangible and qualitative must be evaluated on the basis of executive judgment and intuition.

The fact that operations research is scientific in character rather than expert means that more time is normally required to arrive at any solution or conclusion than in the case of normal engineering analyses.

In conclusion, the future of operations research appears bright. Successful applications in many fields of endeavor and especially industry are overcoming the skepticism of many people. The areas of application of the science is broad. The new and relatively untested theory of games hold promise for the development of strategic concepts. Operations research can and will live up to its expectations of helping executives to make decisions.

CHAPTER VI

CONCLUSION

In the preceding chapters we have discussed statistical method and its relation to the comptroller along with a brief discussion of the historical background concerning the development of mathematical procedures for decision making. The statistical methods and case studies were of necessity brief, but it is hoped that they served to illustrate the various methods of application in solving typical management problems. Perhaps some enthusiasm has been engendered for the science of mathematical decision making. A word of caution is in order, however, these mathematical methods and especially those based on probability theory are only a whetstone for common sense in their applications to problems of the comptroller and executive decision making. Administrative methods already in use within the organization are sharpened with this scientific tool which enable the user to locate and to make the most of significant factors that bear on the problem under analysis. It is appropriate to emphasize that there is no substitute for knowledge, not necessarily knowledge derived from books or a college education, but, empirical knowledge of one's own business. For example, the small business man is better able to predict his firm's sales for the coming

year than a college trained statistician who is unfamiliar with the firm or industry. The statistical formulas presented in this paper cannot evaluate nor recognize and give cognizance to the multitude of factors which bear on the future. This is not to say that statistical formulas are useless when a multitude of data are available. On the contrary, statistical methods are often the only means of determining some order out of unordered and unrelated facts. It, therefore, appears evident that an intelligent forecast based upon sound analysis of the best available data is far superior to a forecast based upon hunches and rumors.

How Much Statistics Should the Comptroller Know?

The comptroller if he is to have an adequate appreciation of the use of statistical method in decision making should acquaint himself with the following broad categories of the statistical field, namely:

1. Sources of Information
2. Decision Making Tools
3. Forecasting
4. Administrative Controls
5. Presentation of Data

Sources of Information

The comptroller, for an adequate understanding of this phase should:

1. Understand fully the problem for which he seeks information.

2. Be able to determine what kinds of data are necessary for the utilization of decision processes.

3. Be aware of the limitations of data obtained within his own organization.

4. Be familiar with listings of source material such as specialized governmental and private publications.

Decision Making Tools

The comptroller should develop an adequate vocabulary in the field of statistical decision theory in order that he may communicate more effectively with his statistical staff. He should understand the appropriateness of the methods being employed in solving a particular problem. The comptroller should have some understanding of the formulas involved in the process, and more important, of the time involved in the application of these formulas. He should be familiar with modern computing techniques in order that he may know where to turn for help and guidance.

Forecasting

Planning is a fundamental problem of the comptroller, and planning involves projection into the future. All forecasting involves some degree of uncertainty. With adequate statistical knowledge, the comptroller can reduce his "uncertain stabbing" decisions and base each more on a sound scientific basis with established and predetermined risks.

A good example of the problem encountered in forecasting is given by Mark Twain when he summed up the nonsense side of extrapolation in his book "Life on the Mississippi:"

> In the space of one hundred and seventy six years the lower Mississippi has shortened itself two hundred and forty-two miles. That is an average of a trifle over one mile and a third per year. Therefore, any calm person, who is not blind or idiotic, can see that in the old oolitic Silurian period just a million years ago next November, the lower Mississippi river was upward of one million three hundred thousand miles long, and stuck out over the Gulf of Mexico like a fishing rod. And by the same token any person can see that seven hundred and forty-two years from now the lower Mississippi will be only a mile and three quarters long, and Cairo (Illinois) and New Orleans will have joined their streets together, and be plodding comfortably along under a single mayor and mutual board of aldermen. There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact.

Administrative Controls

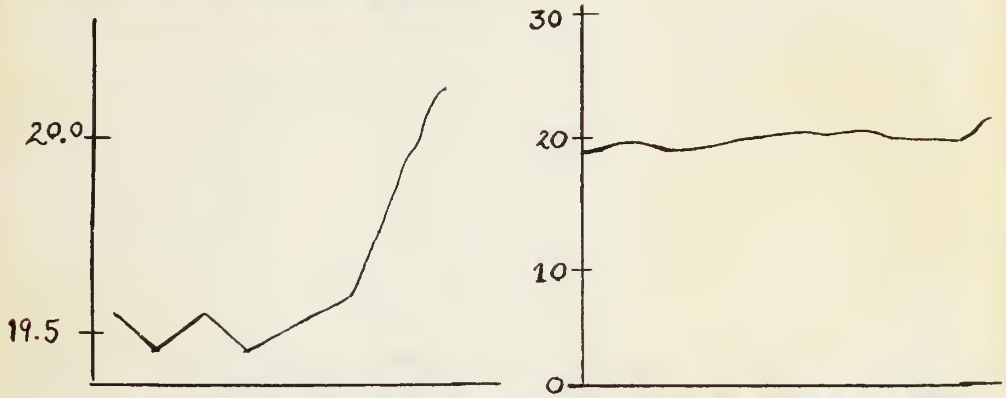
Concerning administrative controls it is a well known fact that these controls are of little value if they are not carried out. The comptroller must have sufficient knowledge of statistical control techniques to be able to interpret changes in control on an objective basis. He must have an understanding of the role played by probability in the control situation, and be able to evaluate departures from control and to take appropriate action when control is violated.

Presentation of Data

The best techniques in decision making, forecasting and control, are of little value if the results are not adequately presented. The comptroller should select tabular and graphic devices which most effectively communicate the desired message. To illustrate this point an editorial writer of a financial magazine reproduced a chart from an advertisement in Washington, D. C. The argument stated Government Payrolls Up! Figures on the graph showed an increase from

19.5 million to 20.2 million. By rescaling the graph an entirely different version was given by this magazine

Government Payrolls stable!



To summarize, it can be seen that the well versed decision making comptroller must have a vast fund of knowledge available to him. Much of this knowledge empirical from a particular field. The comptroller must have the knowledge at his finger tips or know where to get it. He need not be proficient in the deep, mystical, and theoretical foundations of statistics; however, he needs to know enough of the surface material not to be misled or confused. The comptroller must trust the foundations on which his decision-making processes are based, for then only will he have faith in the course of action he chooses to take.

VI APPENDIX

A TABLES

Table I

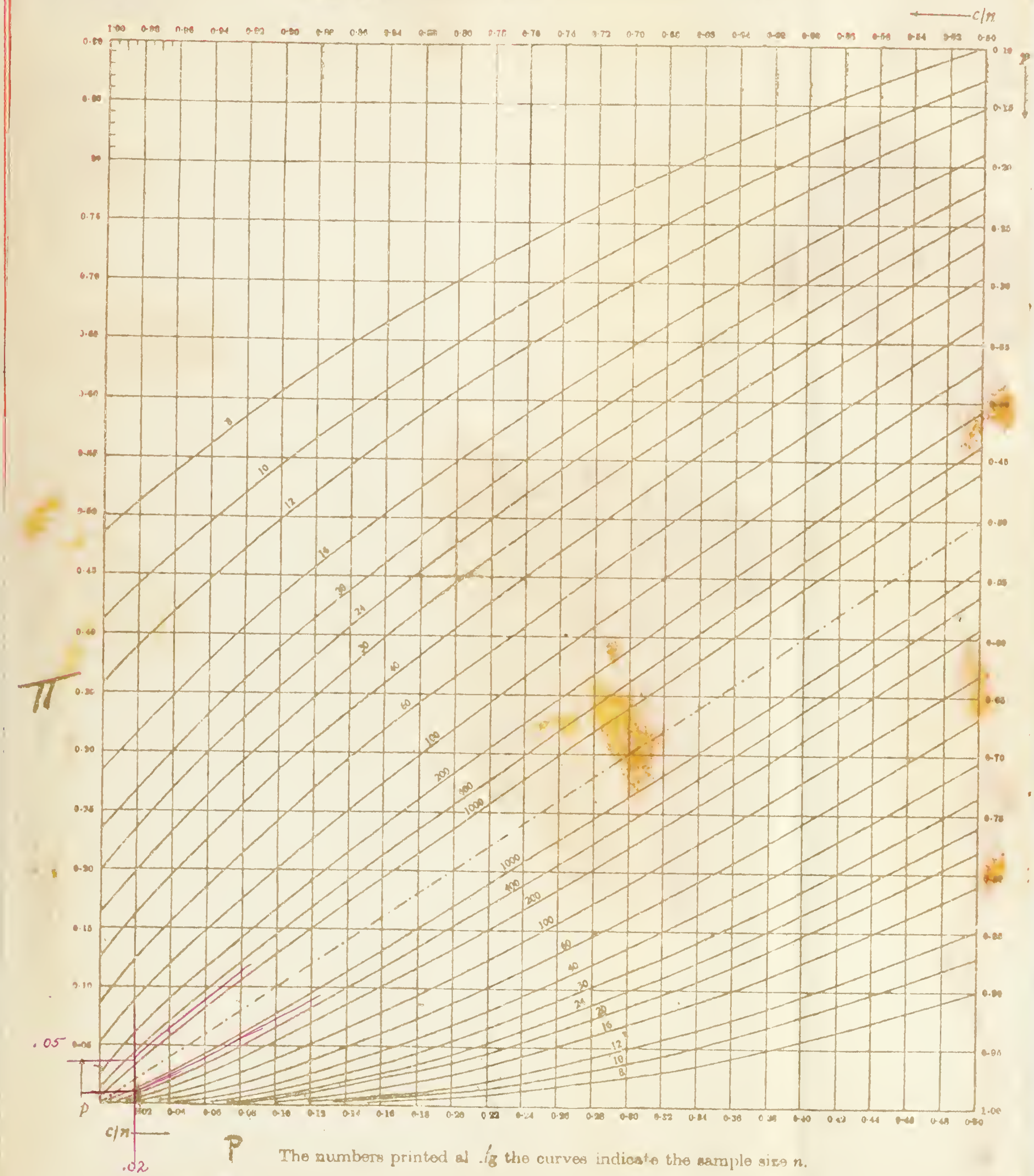
Selected Normal Probabilities

Z	% Area
.126	.05
.253	.10
.385	.15
.524	.20
.6745	.25
.842	.30
1.036	.35
1.282	.40
1.645	.45
1.960	.475
2.326	.49
2.576	.495
3.090	.499

99% Proportions

TABLE II

XXXXXX (*continued*) *Confidence coefficient*, $1 - 2\alpha = 0{\cdot}99$.



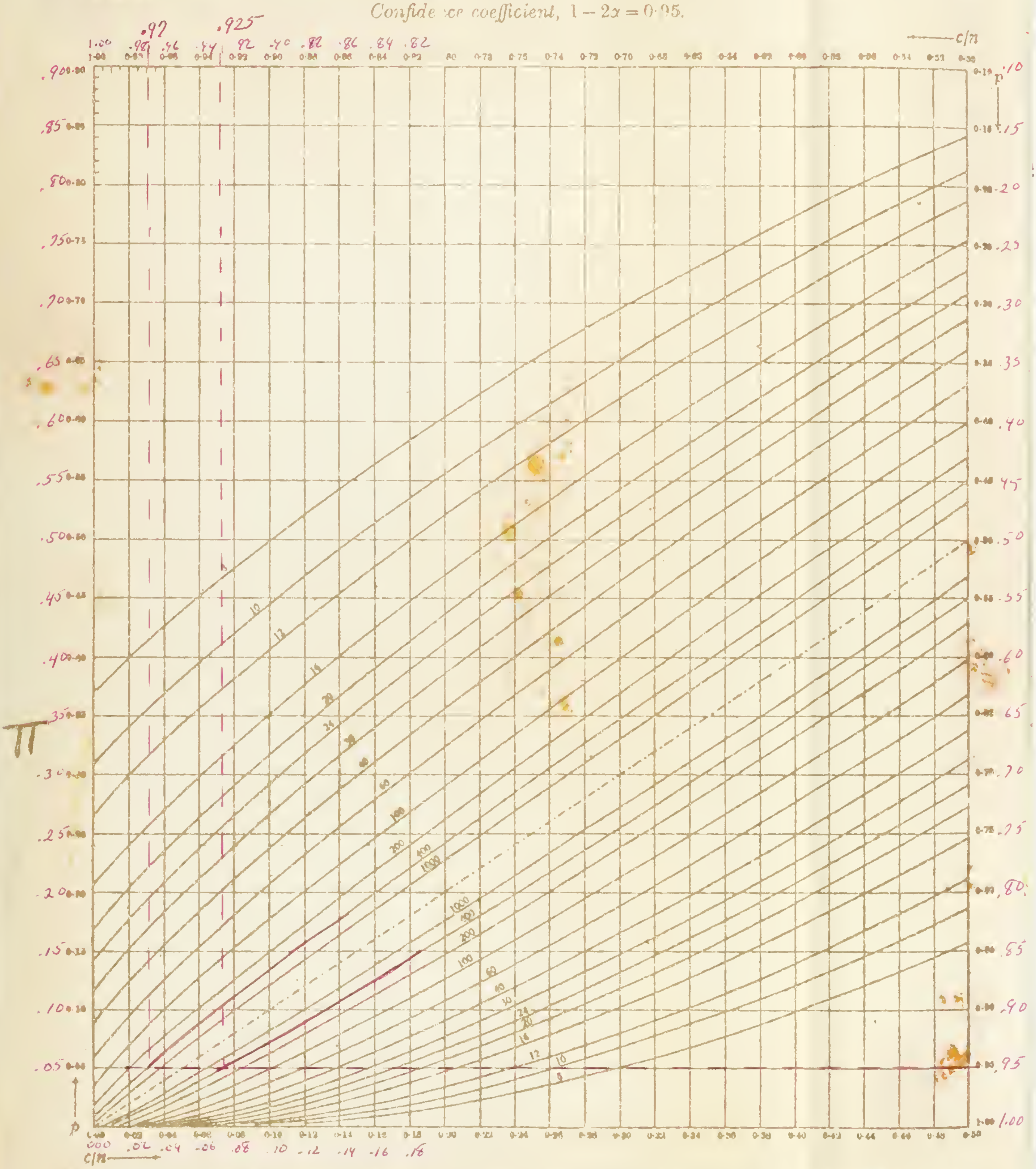
The numbers printed along the curves indicate the sample size n.

Note: the process of reading from the curves can be simplified with the help of the right-angled corner of a loose sheet of paper or thin card, along the edges of which are marked off the scales shown in the top left-hand corner of each Chart.

 95 %

TABLE III

XXXXXX *Chart providing confidence limits for p in binomial sampling, given a sample fraction c/n.*
Confidence coefficient, $1-2\alpha = 0{\cdot}95$.



The numbers printed along the curves indicate the sample size n. If for a given value of the abscissa c/n, p_A and p_B are the ordinates read from (or interpolated between) the appropriate lower and upper curves, then

$$\Pr\{p_A \leq p \leq p_B\} \leq 1-2\alpha.$$

P

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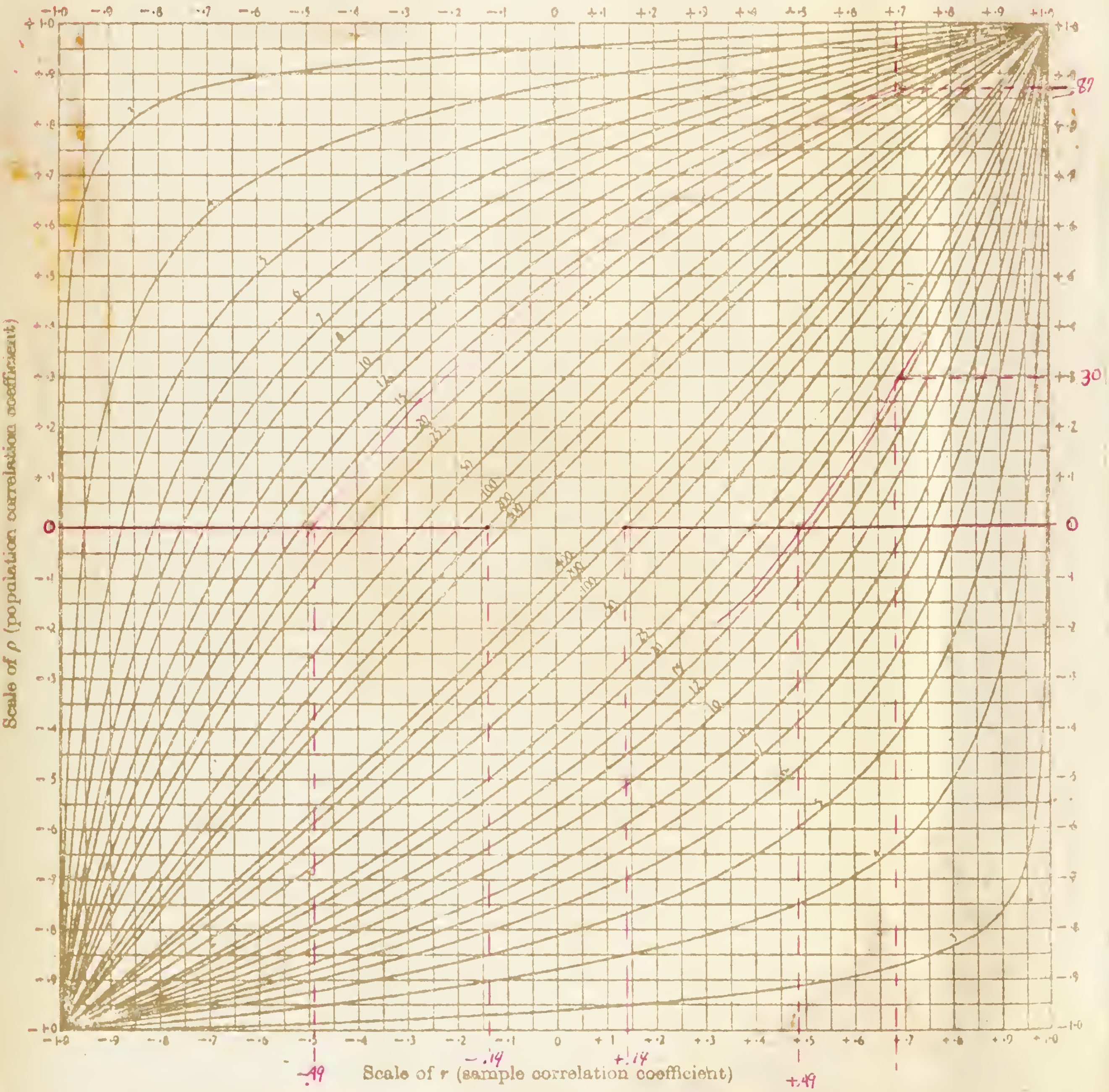
This table is reproduced from E. S. Pearson and H. O. Hartley, "Biometrika Tables for Statisticians", Vol I (1954), by permission of Professor E. S. Pearson.

95 %

TABLE IV

XXXXXX *Chart giving confidence limits for the population correlation coefficient, ρ, given the sample coefficient, r. Confidence coefficient, $1 - 2\alpha = 0{\cdot}95$*



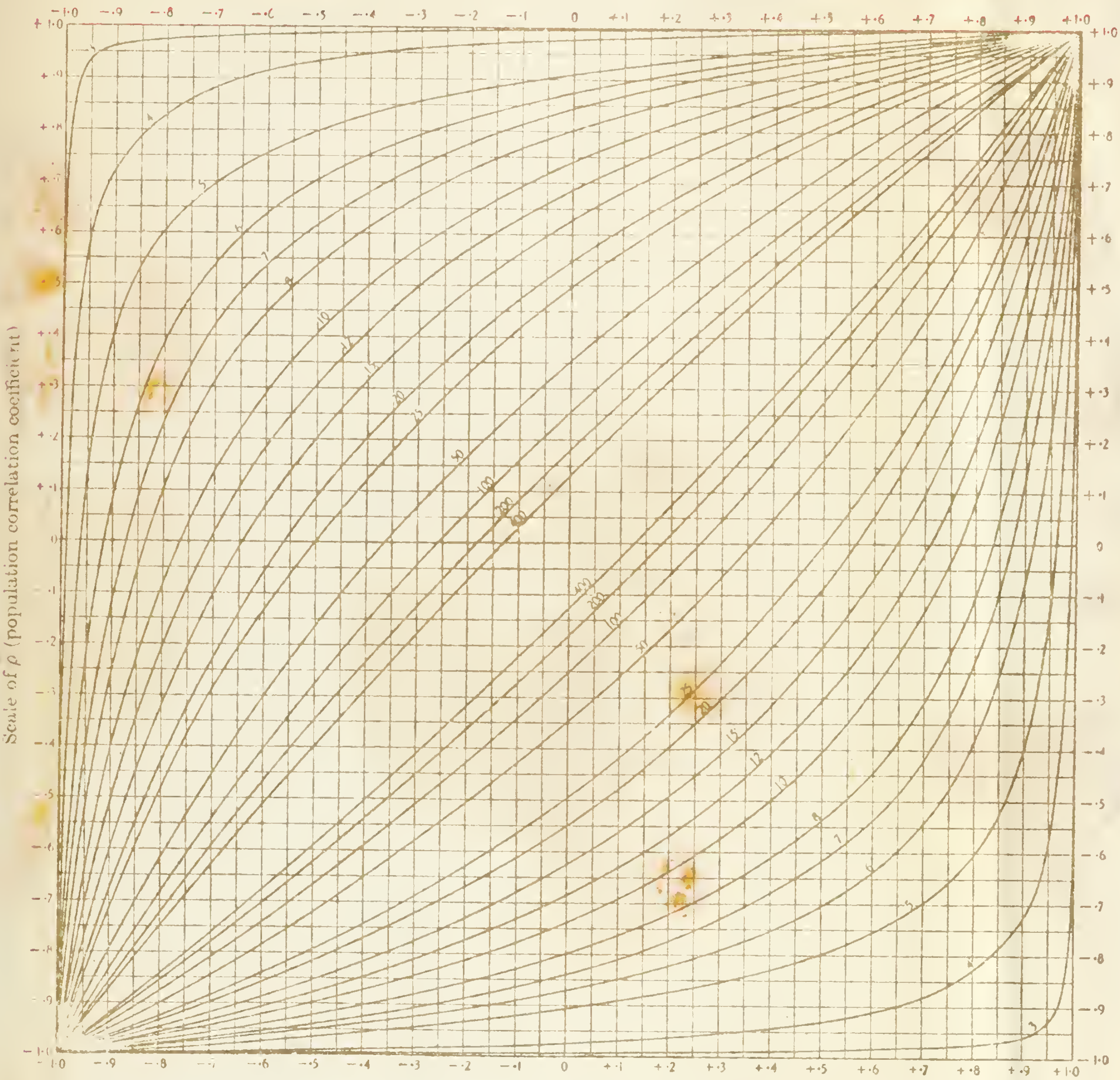
The numbers on the curves indicate sample size. The chart can also be used to determine upper and lower 2·5 % significance points for r, given ρ.

This table is reproduced from "Biometrika Tables For Statisticians" Vol. I (1954), edited by E. S. Pearson and H. O. Hartley, by permission of Professor E. S. Pearson.

99 %

TABLE V

XXXXXX (*continued*). *Confidence coefficient*, $1-2\alpha=0{\cdot}99$



The numbers on the curves indicate sample size. The chart can also be used to determine upper and lower 0·5 % significance points for r, given ρ.

APPENDIX B

Directions for Computing the Mean and Standard Deviation From a Frequency Distribution

Distribution:	X	f	fX	d	fd	fd^2
60 - 64	62	3	186	-3	-9	27
65 - 69	67	8	536	-2	-16	32
70 - 74	72	17	1224	-1	-17	17
75 - 79	77	27	2079	0	0	0
80 - 84	82	25	2050	+1	+25	25
85 - 89	87	20	1740	+2	+40	80
Totals		100	7815		+23	181

X = The mid point between the numbers in the distribution.

f = The frequency or number times these observations occur in the distribution.

fX = Frequency times the mid point.

d = Deviations from the mean.

fd = Frequency times the deviations from the mean.

fd^2 = d times fd.

Σ = Means to sum or add up.

To compute the mean:

$$\text{Mean} = \frac{\Sigma fX}{n} = \frac{7815}{100}$$

$\overline{X} = 78.15$ $\overline{X}$ = Mean. n = number of observations in the distribution

APPENDIX B (CONTINUED)

To compute the Standard Deviation:

S = Standard Deviation and i = the class interval of the distribution 60 to 65 to 70 etc.

$$S = i\sqrt{\frac{fd^2}{n} - \left(\frac{fd}{n}\right)^2}$$

$$S = 5\sqrt{\frac{181}{100} - \left(\frac{+23}{100}\right)^2}$$

$$S = 5\sqrt{1.81 - .0529}$$

$$S = 5\sqrt{1.757100}$$

$$S = 6.625$$

APPENDIX C

Directions for Solving Problems by the Transportation Problem Procedure

This sample consists of assigning the manner of shipping railroad empties from three shipping points having excess number of empties to five destinations having, in total, the same number of freight car shortages. The data for the problem is given in Table A below; that is, the shipping rates from each shipping point to each destination, the excesses at each shipping point, and the shortages at each destination.

(A)	D1	D2	D3	D4	D5	Surplus Totals
S1	$10	20	5	9	10	9
S2	2	10	8	30	6	4
S3	1	20	7	10	4	8
Shortage Totals	3	5	4	6	3	21

It is first necessary to get a feasible solution, that is, one which meets the fixed requirements (of excesses and shortages) regardless of cost. This solution may be obtained as follows. Take the 9 cars at shipping point S1 and fill the shortages of 3 cars at D1 and 5 at D2; put the remaining car at D3. Take the 4 cars at S2 and put 3 more at D3 to complete the shortage there of 4, and put the fourth car at D4. Take the 8 cars at S3, put 5 at D4 to complete the shortage there of 6 and put the remaining 3 at D5, which fills the shortage there of 3. The procedure could obviously be used to fill a table of any size. This solution is entirely feasible and would cost $351.
(3 x $10 + 5 x $20 + 1 x $5 + 3 x $8 + 1 x $30 + 5 x $10 + 3 x $4 = $351)
The table would then be shown in Table B.

(B)	D1	D2	D3	D4	D5	
S1	3	5	1			9
S2			3	1		4
S3				5	3	8
	3	5	4	6	3	21

Since we learned in Chapter 4 that the method we are using leads to the optimum solution by proceeding through a series of solutions, each closer to the optimum than its predecessor, and never repeating, it is obvious that we may

save ourselves a number of computational steps if we obtain a first solution which is the most economical we can get by quick inspection. Stated differently, if we use common sense to get a starting point which is fairly inexpensive in terms of shipping costs, we will save ourselves a great deal of computational effort, even in a problem as short and simple as the example we are using for instruction purposes. Therefore let us attack the problem as follows. Take any shipping point at all and use the capacity (number of empties) there to fill those destination requirements (shortages) which by the cost of shipping it seems most economical to fill. When that capacity is exhausted, take another shipping point and do the same, first filling any requirements that were only partially satisfied by the preceding shipping point. Using this procedure we can proceed as follows. Take the 8 cars at S3 and put 3 at D1 at \$1 each; put 3 of the remaining at D3 at \$4 each; and the other 2 at D3 at \$7 each. Fill D3's remaining requirements with 2 cars from S2, and put the remaining 2 from S2 at D2 (both D1 and D5 cost less but are already full from the shipment from S3). Finally, complete the table by placing the 9 cars from S1, 3 at D2 and 6 at D4. We then have Table C, at a cost of \$179, which is considerably better than our random solution of Table B, and therefore offers us fewer steps to go through to achieve the optimum solution.

(C)	D1	D2	D3	D4	D5	
S1		3		6		9
S2		2	2			4
S3	3		2		3	8
	3	5	4	6	3	

It is next necessary to build up a type of cost table. First, fill in a table showing the actual shipping costs, taken from Table A, for those shipments which are actually in use in Table C. This gives Table D. We can refer to the "squares" of the table as "square S1D1" etc., and in Table D, S1D4 is equal to \$9, or S1D4 = 9.

(D)	D1	D2	D3	D4	D5
S1		20		9	
S2		10	8		
S3	1		7		4

Second, in building up the cost table, of which Table D is the first step, assign row and column values on an arbitrary basis; row and column values are the values shown under those headings in Table E. They are obtained by assigning a value to row S1, such as zero, or 1, or 2, or anything (we have chosen zero) and then under every square of row S1 which contains a rate (S1D2, S1D4, etc.) assign a column value such

that the sum of the row and the column values equals the sum of the value in the table. Thus, we obtain Table E, in which S1D2 is 20, and since the row value was zero, the column value must be 20; S1D4 is 9 and therefore the column value under D4 must be 9 so that the row value zero and the column value 9 equal the square S1D4. It should be noted that both positive and negative values of row and column values may occur. In observing S2D2 we see that we already have a column value of 20, and that the square value is 10, and therefore the row value for S2 must be -10. We then proceed to determine the column value for D3, since we have a row value of -10 and a square value of 8 for S2D3; the column value is necessarily 18. Table E is completed by similar steps to give the values shown.

(E)	D1	D2	D3	D4	D5	Row Values
S1		20		9		0
S2		10	8			-10
S3	1		7		4	-11
Column Values	12	20	18	9	15	

Finally, we make Table E into a cost table, Table F, by filling in all the blank squares with the sum of the developed row and column values as they are shown in Table E. S1D1 then becomes 12, the sum of the S1 value of Zero and the D1 value of 12, etc.

(F)	D1	D2	D3	D4	D5	Row Values
S1	12	20	18	9	15	0
S2	2	10	8	-1	5	-10
S3	1	9	7	-2	4	-11
Column Values	12	20	18	9	15	

As one develops familiarity with the method, he omits the majority of the tables which have been added for instructional purposes. All that is needed is a rate table, Table A; a route table, Table C; and a cost table, Table F, for the first stage of the method.

It is now possible to determine what change in our route of Table C should be made to reduce our shipping cost. Examine the table in comparison to the rate table, and find the square where the figure in Table F is larger by the greatest amount than the figure in Table A. By inspection this is square S1D3 in which we find a value of 18 compared to Table A's value of 5. We now know that a change in the route so that more cars go from S1 to D3 we will save $13 on each additional car that we can put on this route. A detailed explanation of the reason for this statement may be found in Henderson and Schlaifer's article. The problem is now to find out how many cars we can shift to square D1D3, i.e., how many

cars we can send from shipping point S1 to destination D3 instead of using the solution of Table C. We do so as follows. Construct Table G, and in square S1D3 write +x: this is the unknown amount which will be shipped from S1 to D3. The other amounts in Table G are obtained by copying Table C.

(G)	D1	D2	D3	D4	D5	
S1		3	+x	6*		9
S2		2	2			4
S3	3*		2		3*	8
	3	5	4	6	3	

But we must know which routes are unaffected if we are to complete the table properly. We mark those squares in Table G, after copying the Table C values, with an * or some mark, which is the <u>only</u> number in either its row or its column. The +x in S1D3 is counted as a number for this purpose. Hence we mark the value S3D1 with an *, since 3 is the only number in the row D1; and we mark the 6 of S1D4 and the 3 of S3D5 with an *. We next consider the * numbers as not in the table, and go through the * procedure again, looking for any numbers which are now alone in either rows or columns. Thus we * square S3D3, where the 2 is the only number meeting the requirement of being alone, in this case in row S3. We cannot * any more numbers. We now have Table H, except that the only square with an x value in it is the +x in S1D3. We complete Table H by disregarding all the * values and filling in +x and -x values such that the rows and columns add up properly to the totals shown, S1D2 must necessarily become 3-x so that the sum of the three values of row S1 (3-x, +x, 6) add up to the known total 9.

(H)	D1	D2	D3	D4	D5	
S1		3-x	+x	6*		9
S2		2+x	2-x			4
S3	3*		2*		3*	8
	3	5	4	6	3	

The maximum number of cars we can divert to S1D3 is then 2, for if we diverted any more we should have a minus number of cars for square S2D3, where our new value is 2-x; one of the postulates of our problem is of course that we do not want negative shipments, i.e., shipments from destinations to shipping points. This restraint is normally referred to as a "non-negativity" restraint. By substituting the value of 2 for x, we obtain Table J (Table I omitted) and can determine that the shipping cost is now only \$154, or an improvement over our value of \$179; and as we said, we save 2 cars at \$13 each, or \$26 over the first solution, or \$179 - \$26 = \$154.

(J)	D1	D2	D3	D4	D5	
S1		1	2	6		9
S2		4				4
S3	3		2		3	8
	3	5	4	6	3	

It is now necessary to repeat the same procedure, beginning with Table J, as we have just done in beginning with Table C. Tables K and L are intermediate steps to obtaining Table M. In Table K, a row value of 10 for row S1 was taken, and the complete table developed just as was done for Table F. Comparison of Table K with Table A shows that S3D2 in the former is higher in value by 2, and therefore that we will save $2 for each car that we can ship over that route if we make the proper adjustments in the other routes. Table L, developed just as was Table H, shows that we can ship 1 car over route S3D2. Our new routing table is then Table M, and its cost is $151, or $2 less than our previous routing Table J.

(K)	D1	D2	D3	D4	D5	
S1	-1	20	5	9	2	10
S2	-11	10	-5	-1	-8	0
S3	1	22+2	7	11+1	4	12
	-11	10	-5	-1	-8	

(L)	D1	D2	D3	D4	D5		
S1		1-x	2+x	6*		9	
S2		4*				4	
S3	3*	+x	2-x		3*	8	x = 1
	3	5	4	6	3		

(M)	D1	D2	D3	D4	D5	
S1			3	6		9
S2		4				4
S3	3	1	1		3	8
	3	5	4	6	3	

Once more we go through the procedure, obtaining Tables N and O in reaching Table P, and improve our cost to $150.

(N)	D1	D2	D3	D4	D5	
S1	-1	18	5	9	2	9
S2	-9	10	-3	1	-6	1
S3	1	20	7	11+1	4	11
	-10	9	-4	0	-7	

(O)	D1	D2	D3	D4	D5	
S1			3+x	6-x		
S2		4*				x=1
S3	3*	1*	1-x	+x	3*	

(P)	D1	D2	D3	D4	D5
S1			4	5	
S2		4			
S3	3	1		1	3

Another trial, giving Table Q, shows that in no case is there any value in Table Q which is higher than its corresponding square in Table A. We know then that we have, in Table P, reached the optimum arrangement. There remain no changes we can make in that table to obtain a lower cost of making the shipments of empty freight cars that Table A gave as being required. We have given the freight car dispatcher the information on which to make his decision; should he decide, for reasons best known to him, to make a shipment other than Table P shows, he can compute the excess cost to do so, and base his decision on concrete cost data, not conjecture.

(Q)	D1	D2	D3	D4	D5	
S1	0	19	5	9	3	5
S2	-9	10	-4	0	-6	-4
S3	1	20	6	10	4	6
	-5	14	0	4	-2	

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GLOSSARY

Arithmetic mean - the simple average of a set of observations. It is obtained by adding a set of observations and dividing by the number of observations.

Average (mean) deviation - the simple average of the absolute distances of the observations from the mean.

Binomial distribution - a distribution in which there are only two alternatives. Sometimes referred to as a dichotomous distribution. For example, light bulbs are defective or good.

Binomial expansion - raising the binomial to a given power.

Binomial theorem - the theorem by means of which a binomial may be raised to any power without performing the multiplications. A binomial is an expression consisting of two terms connected by a plus or a minus sign. A binomial distribution is a distribution where there are only two alternatives. For example in sampling light bulbs, they are either good or bad.

Coefficient of variation - a measure indicating the percent the standard deviation is of the arithmetic mean.

Correlation coefficient - an index measuring the extent of relationship between two variables. It ranges from -1 (a perfect negative relationship) to 1 (a perfect positive relationship).

Estimation - that phase of inference in which sample characteristics are used to provide estimates of corresponding population characteristics.

Hypothesis - a statement of fact which it is desired to test. Usually referred to as a null hypothesis when testing a specified characteristic of a population.

Inter-quartile range - the middle 50% of the observations. Those two values which include the middle 50% of the cases in a distribution.

Median - The middle observation when a set of data have been ordered or ranked.

Mode - that observation in a distribution which occurs more than any other.

Normal curve - a symmetrical bell shaped curve possessing certain statistical characteristics.

Normal distribution - often referred to as Gaussian. A bell shaped curve possessing certain statistical characteristics.

Parameter - a characteristic in a population. For example, the true proportion of defectives in a lot of 100,000 shells.

Poisson distribution - a skewed distribution which is encountered frequently in supply problems.

Population - sometimes referred to as universe. The entire group or set of observations in which we have an interest.

Proportion - a measure between 0 and 1.00 indicating the ratio of the number possessing a given characteristic to the total number of observations. For example, a proportion of .80 means that 80% of the observations possess a certain characteristic.

Random - a procedure whereby only chance influences selection. Not synonymous with haphazard.

Range - the distance between the largest and smallest observation in a set of observations.

Regression - a study of the predictive value of a line in explaining the degree of relationship between two variables.

Regression coefficient - the average amount of change in Y per unit change in X.

Sample - a part, or sub-set, of a population or universe.

Skewness - a departure from symmetry.

Standard deviation - the most reliable measure of variation. Often referred to as the root-mean-square.

Standard error of estimate - a measure of the scatter about the line of regression. The standard deviation of the observed points about the line.

Standard error of the mean - the standard deviation of sample means. A measure of the variability in a sample mean.

Standard score - the number of standard deviations an observation is from the arithmetic mean. For example, a standard score of -2 indicates that the observation is two standard deviations below the mean.

Statistic - a characteristic in a sample. For example, the proportion of defectives in a sample of 100 shells.

Statistics - that branch of applied mathematics which deals with experimental design and the collection, tabulation, analysis, interpretation, and presentation of data.

T scale - a theoretical distribution that goes by the name of Student-T distribution, developed by W. S. Gosset who published his work under the pen name of Student.

Theory of least squares - mathematically computed lines may be passed through the data, or scatter points, which are called regression lines, or lines of average relationship, because they reveal the typical change in the dependent variable Y which has, in the past, accompanied a given change in the variable plotted on the X axis. This average relationship may be determined mathematically by the method of least squares.

Type I error - rejecting a hypothesis when in fact it is true.

Type II error - accepting a hypothesis when in fact it is false.

Variable - that which does not maintain a constant value. The existence of variables brings forth the need for statistics.